ANSWERS

1.01

1 **a** $8x^3 + 9x^2$ **b** $12x - 1$ **c** $30x + 21$
d $72x^5 - 16x^3$ **e** $30x^4 - 4x$

2 **a** $12x + 1$ **b** $15x^2 + 4x - 5$
c $5(x^3 + 2x - 1) + (5x - 3)(3x^2 + 2)$
d $2x(x^2 - x - 1) + (2x - 1)(x^2 + 7)$
e $3x^2(x^4 + 2x^3 - 5x^2 + x - 2) +$
$(x^3 + 3)(4x^3 + 6x^2 - 10x + 1)$

3 **a** $\dfrac{-2}{(2x-1)^2}$ **b** $\dfrac{15}{(x+5)^2}$
c $\dfrac{x^4 - 12x^2}{(x^2-4)^2} = \dfrac{x^2(x^2-12)}{(x^2-4)^2}$
d $\dfrac{16}{(5x+1)^2}$ **e** $\dfrac{-x^2+14x}{x^4} = \dfrac{-x+14}{x^3}$
f $\dfrac{11}{(x+3)^2}$ **g** $\dfrac{-2x^2}{(2x^2-x)^2}$
h $\dfrac{-6}{(x-2)^2}$ **i** $\dfrac{-34}{(4x-3)^2}$
j $\dfrac{-14}{(3x+1)^2}$

4 **a** $-4x^{-5}$ **b** $-8x^{-9}$ **c** $-6x^{-4}$
d $-55x^{-12}$ **e** $-\dfrac{9x^{-10}}{5}$ **f** $\dfrac{1}{2}x^{-\frac{1}{2}}$ **g** $\dfrac{1}{4}x^{-\frac{3}{4}}$
h $\dfrac{3}{7}x^{-\frac{6}{7}}$ **i** $\dfrac{10}{3}x^{-\frac{1}{3}}$ **j** $-x^{-\frac{3}{2}}$

5 **a** $-\dfrac{5}{x^6}$ **b** $-\dfrac{6}{x^7}$ **c** $-\dfrac{6}{x^4}$ **d** $-\dfrac{2}{3x^3}$
e $-\dfrac{4}{5x^2}$ **f** $\dfrac{1}{2\sqrt{x}}$ **g** $\dfrac{1}{6\sqrt[6]{x^5}}$ **h** $\dfrac{4}{3\sqrt[3]{x^2}}$
i $\dfrac{2}{3\sqrt[3]{x}}$ **j** $\dfrac{5\sqrt{x^3}}{2}$

6 **a** $-\dfrac{1}{16}$ **b** -3 **c** 852
d $26\dfrac{65}{96}$ **e** -320

7 **a** 160 **b** $-\dfrac{1}{9}$ **c** -5

8 **a** $x < 0$ **b** $x > \dfrac{1}{\sqrt{27}}$ or $x < -\dfrac{1}{\sqrt{27}}$ **c** $x > 0$
d $-5 < x < -1$ **e** None

9 $(1, 1)$ and $\left(-\dfrac{5}{9}, -1\dfrac{32}{243}\right)$

10 $x = -8, 2$

11 $17x - 25y - 19 = 0$

12 **a** \$564 816 **b** **i** \$84 755.20 **ii** \$376 960

13, 14 Proofs

1.02

1 **a** $m^2, 3x + 3, (3x + 3)^2$
b $3g + 3, x^2$ and $3x^2 + 3$
c $(3x + 3)^2 + 4 = 9x^2 + 18x + 13$
d $3(x^2 + 4) + 3 = 3x^2 + 15$
e $\sqrt[3]{x^2 + 4}$ **f** $\sqrt[3]{x^2} + 4$
g 9 **h** 3 **i** 169 **j** 85

2 **a** $4(x + 3)^3$ **b** $6(2x - 1)^2$
c $70x(5x^2 - 4)^6$ **d** $48(8x + 3)^5$
e $-5(1 - x)^4$

3 **a** $135(5x + 9)^8$
b $4(x - 4)$
c $4(6x^2 + 3)(2x^3 + 3x)^3$
d $8(2x + 5)(x^2 + 5x - 1)^7$
e $12(3x^5 - 2x)(x^6 - 2x^2 + 3)^5$

4 **a** $\dfrac{3}{2}(3x-1)^{-\frac{1}{2}}$ **b** $2(4 - x)^{-3}$
c $-6x(x^2 - 9)^{-4}$ **d** $\dfrac{5}{3}(5x+4)^{-\frac{2}{3}}$
e $\dfrac{3}{4}(3x^2 - 14x + 1)(x^3 - 7x^2 + x)^{-\frac{1}{4}}$

5 **a** $\dfrac{3}{2\sqrt{3x+4}}$ **b** $-\dfrac{5}{(5x-2)^2}$
c $-\dfrac{8x}{(x^2+1)^5}$ **d** $-\dfrac{2}{\sqrt[3]{7-3x}}$
e $-\dfrac{5}{2\sqrt{(4+x)^3}}$

6 **a** $x(x + 1)^2(5x + 2)$ **b** $8(9x - 1)(3x - 2)^4$
c $3x^3(16 - 7x)(4 - x)^2$ **d** $(10x + 13)(2x + 5)^3$
e $10x(x^3 + 5x^2 - 3)(x^2 + 1)^4 + (3x^2 + 10x)(x^2 + 1)^5$

7 **a** $\dfrac{6(5x+1)(2x-9)^2 - 5(2x-9)^3}{(5x+1)^2}$
$= \dfrac{(2x-9)^2(20x+51)}{(5x+1)^2}$
b $\dfrac{(7x+2)^4 - 28(x-1)(7x+2)^3}{(7x+2)^8} = \dfrac{-21x+30}{(7x+2)^5}$
c $\dfrac{15(2x-5)^3(3x+4)^4 - 6(3x+4)^5(2x-5)^2}{(2x-5)^6}$
$= \dfrac{3(3x+4)^4(4x-33)}{(2x-5)^4}$

d $\dfrac{3\sqrt{x+1}-\dfrac{3x+1}{2\sqrt{x+1}}}{x+1}=\dfrac{3x+5}{2\sqrt{(x+1)^3}}$

e $\dfrac{\dfrac{2x-3}{2\sqrt{x-1}}-2\sqrt{x-1}}{(2x-3)^2}=\dfrac{-2x+1}{2\sqrt{x-1}(2x-3)^2}$

8 **a** 245 **b** $-\dfrac{152}{2401}$ **c** $\dfrac{20\sqrt[3]{25}}{3}$

d $-\dfrac{12\,544\sqrt[3]{25}}{15}$ **e** $-\dfrac{4}{27}$

9 (1, 1)

10 **a** 2.2×10^{14} L **b** 3.1×10^{11} L/min

c 2.7×10^{13} L/min

11 **a** $x\geq4$ **b** $\dfrac{1}{6}$ **c** $y=\dfrac{1}{6}x+\dfrac{5}{6}$

d $\dfrac{1}{2\sqrt{a-4}}$ **e** $\dfrac{\sqrt{a-4}}{a}$ **f** (8, 2)

12 **a** −4 **b** −4 **c** Parallel **d** −25

e −25 **f** Parallel **g** −1 **h** −1

i Parallel **j** $x=-2\pm k$

13, 14 Proofs

INVESTIGATION: DERIVATIVES OF a^x

1

x	0.5	0.8	1.5	2	2.5
$\lim\limits_{h\to0}\dfrac{a^h-1}{h}$	−0.6931	−0.2231	0.4055	0.6931	0.9163
x	3	3.5	4	5	
$\lim\limits_{h\to0}\dfrac{a^h-1}{h}$	1.0986	1.2528	1.3863	1.6094	

2 $\dfrac{d}{dx}(a^x)\approx-0.6931a^x$ for $x=0.5$

$\dfrac{d}{dx}(a^x)\approx-0.2231a^x$ for $x=0.8$

$\dfrac{d}{dx}(a^x)\approx0.4055a^x$ for $x=1.5$

$\dfrac{d}{dx}(a^x)\approx0.6931a^x$ for $x=2$

$\dfrac{d}{dx}(a^x)\approx0.9163a^x$ for $x=2.5$

$\dfrac{d}{dx}(a^x)\approx1.0986a^x$ for $x=3.5$

$\dfrac{d}{dx}(a^x)\approx1.2528a^x$ for $x=3.5$

$\dfrac{d}{dx}(a^x)\approx1.3863a^x$ for $x=4$

$\dfrac{d}{dx}(a^x)\approx1.6094a^x$ for $x=5$

3 The graph for $a=0.5$ is reflected in the y-axis compared to $a=2$. The graphs the derivatives for $a=0.8$ and $a=0.5$ are below those of the functions for $x<0$.

The graphs for $a=1.5$ and 2 look similar, with the graphs of the derivatives below those of the functions for $x>0$.

The graphs for $a=3$, 3.5, 4 and 5 look similar, with the graphs of the derivatives above those of the functions for $x>0$.

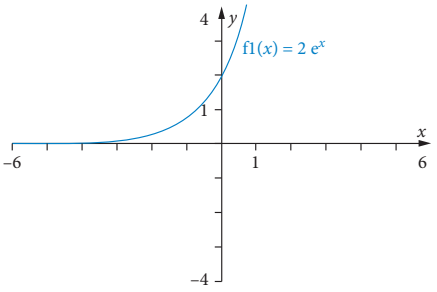
4 As a increases, the graph of the derivative moves from below that of the function to above and further away.

1.03

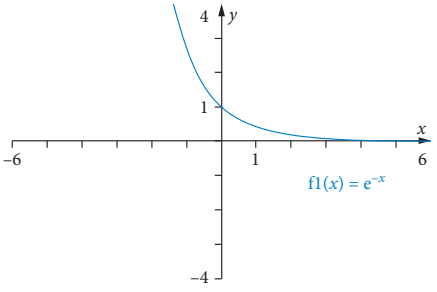
1 **a** 4.48 **b** 0.14 **c** 2.70

d 0.05 **e** −0.14

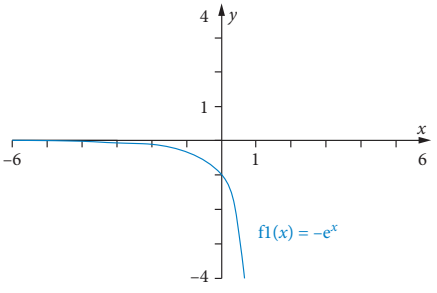
2 **a**



b



c



3 **a** e **b** 1.79 **c** $x-ey+2=0$

4 **a** $9e^x$ **b** $-e^x$

c e^x+2x **d** $6x^2-6x+5-e^x$

e $3e^x(e^x+1)^2$ **f** $-9e^x(5-e^x)^8$

g $12e^x(2e^x-3)^5$ **h** $4(e^x+1)(e^x+x)^3$

5 **a** $3e^{3x}$ **b** $2e^{2x-1}$ **c** $8e^{4x}$

d $2x\,e^{x^2-1}$ **e** $(10x^4-9x^2+1)\,e^{2x^5-3x^3+x-3}$

6 **a** $e^x+xe^x=e^x(x+1)$

b $2e^x+e^x(2x+3)=e^x(2x+5)$

c $15x^2e^x+5x^3e^x=5x^2e^x(x+3)$

d $2e^{3x}+6xe^{3x}=2e^{3x}(3x+1)$

e $2e^{2x}(x^2+x+2)+e^{2x}(2x+1)=e^{2x}(2x^2+4x+5)$

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7 **a** $\dfrac{xe^x - 2e^x}{x^3} = \dfrac{e^x(x-2)}{x^3}$

b $\dfrac{6xe^{6x} - e^{6x}}{3x^2} = \dfrac{e^{6x}(6x-1)}{3x^2}$

c $\dfrac{10xe^{5x} - 6e^{5x}}{5x^4} = \dfrac{2e^{5x}(5x-3)}{5x^4}$

d $\dfrac{e^x - (x-1)e^x}{e^{2x}} = \dfrac{2-x}{e^x}$

e $\dfrac{e^{3x} - 2e^{2x}(e^x+1)}{e^{4x}} = \dfrac{-(e^x+2)}{e^{2x}}$

8 **a** 2.71 **b** $28e^{-4} \approx 0.5128$ **c** 348.4

d $80e^6 + e^2$ **e** $5e^{-6}$, 25

9 2

10 $k = -\dfrac{1}{2}$

1.04

1 **a** **i** 175 cases **ii** 186 cases
iii 254 cases **iv** 877 cases

b **i** 11.5 cases/week **ii** 15.7 cases/week
iii 54.4 cases/week

2 **a** 180 sales **b** 966 sales
c 116 sales/day **d** 3337 sales/day

3 **a** 1100 swans

b **i** 1246 swans **ii** 1485 swans
iii 2706 swans

c **i** 28 swans/month **ii** 31 swans/month
iii 37 swans/month

4 **a** 200 g

b **i** 188.4 g **ii** 157.3 g **iii** 60.2 g

c **i** 2.26 g/year **ii** 1.89 g/year **iii** 0.72 g/year

5 **a** **i** 86 270.8 ha **ii** 52 588.2 ha **iii** 23 046 ha

b **i** 3707.1 ha/year **ii** 2413.9 ha/year
iii 1057.9 ha/year

6 **a** $N = 90\,000e^{0.29t}$ **b** 512 761 bacteria

c 148 701 bacteria/hour **d** 474 345 bacteria/hour

7 **a** $R = 43e^{-0.008t}$

b **i** 39.7 cm **ii** 33.8 cm **iii** 19.3 cm

c **i** 0.32 cm/year **ii** 0.27 cm/year **iii** 0.15 cm/year

8 **a** P_0 **b** 15.5% **c** $\dfrac{dP}{dt} = 0.024P$

9 **a** **i** 87% **ii** 49.7% **iii** 24.7%

b 10 years

10 **a** $2e^4x - y - 3e^4 = 0$ **b** $(1\frac{1}{2}, 0)$

c $\dfrac{e^4}{4}$ units^2

11 **a** 102

b **i** 112 **ii** 2779

c $\dfrac{dP}{dt} = 0.6e^{0.3t}$

d **i** 4 birds/month **ii** 804 birds/month

12 **a** $N_0 = 30\,000$ **b** 12 102 864

c **i** 14 523 437 bacteria/hour
ii 6.5×10^{10} **iii** 1.2×10^{17}

INVESTIGATION: ESTIMATION OF DERIVATIVES OF sin (*x*) AND cos (*x*)

The derivative of sin (x) is close to cos (x).
The derivative of cos (x) is close to $-\sin(x)$.
As h is made smaller, the approximations become more exact.

1.05

1 **a** $1 + \cos(x)$ **b** $6\cos(x)$

c $6\cos(6x)$ **d** $2x\cos(x^2 - 3)$

e $\dfrac{4}{3}\cos\left(\dfrac{x}{3}\right)$

2 **a** $6\cos(x)(\sin x + 9)^5$ **b** $\sin(x) + x\cos(x)$

c $e^x\cos(e^x)$

d $\dfrac{2[x\cos(2x) - \sin(2x)]}{5x^3}$ **e** $\cos(x)\, e^{\sin(x)}$

3 **a** $\dfrac{1}{2}$ **b** $\dfrac{12}{\sqrt{2}} = 6\sqrt{2}$ **c** 0

d $1 + \dfrac{\sqrt{3}}{2} = \dfrac{2+\sqrt{3}}{2}$ **e** $-\dfrac{4}{\pi^2}$

4 **a** $-\sin(x)$ **b** $-3\sin(x)$

c $-5\sin(5x)$ **d** $-6x\sin(3x^2 + 1)$

e $-\sin\left(\dfrac{x}{2}\right)$

5 **a** $3[4 - \sin(x)][4x + \cos(x)]^2$

b $\cos(x) - x\sin(x)$ **c** $-3e^{3x}\sin(e^{3x})$

d $\dfrac{-x\sin(x) - \cos(x)}{3x^2}$ **e** $-\cos(x)\sin[\sin(x)]$

6 **a** $\dfrac{1}{\cos^2(x)}$ **b** $1 + \dfrac{6}{\cos^2(x)}$

c $\dfrac{9}{\cos^2(9x)}$ **d** $\dfrac{12}{\cos^2(4x)}$

e $\dfrac{5}{\cos^2(x)}[\tan(x) - 1]^4$

7 **a** $2x\tan(x) + \dfrac{x^2}{\cos^2(x)}$

b $\dfrac{2x - \tan(2x)\cos^2(2x)}{x^2\cos^2(2x)}$ **c** $\dfrac{e^{\tan(x)}}{\cos^2(x)}$

d $\dfrac{-\sin(x)}{\cos^2[\cos(x)]}$ **e** $\dfrac{1 - \tan(x)\cos^2(x)}{e^x\cos^2(x)}$

8 **a** $e(2\sin(0.5) + \cos(0.5))$ **b** $\dfrac{\pi^2 + 4\pi}{8}$

c $-\dfrac{\sqrt{3}}{2}$ **d** $-\sin(e) - \dfrac{\cos(e)}{e}$

e $3e^2\cos(e^2) - 2e^4\sin(e^2)$

9 **a** 0.157 **b** -0.269

10 $x = \pm\dfrac{\pi}{4}, \pm\dfrac{3\pi}{4}, \pm\dfrac{5\pi}{4}, \pm\dfrac{7\pi}{4}, \ldots$

11 **a** 2 cm **b** $0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \ldots$s

c 0 cm/s **d** $\frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}$s

12 **a** 2.3 mm/s **b** 3 mm/s^2

c Proof

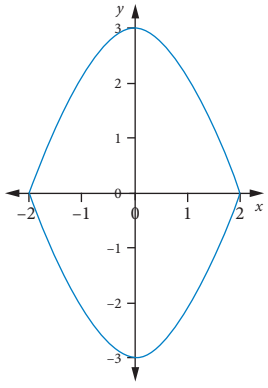
1.06

1 About 1.15^c or 65.9°.

2 About 1.23^c or 70.5°.

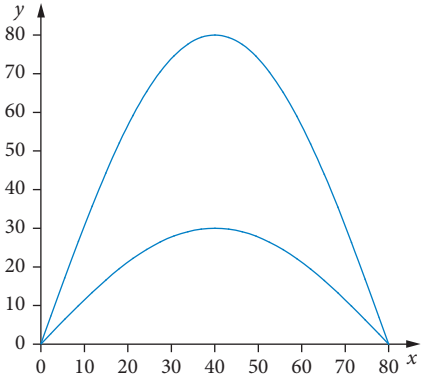
3 About 0.881^c or 50.5°.

4 **a**



b 1 cm **c** About 2.34^c or 134.0°

5 **a**



b 50 m **c** About 0.396^c or 22.7°.

6 **a** $6\sqrt{3}x - 12y + 6 - \sqrt{3}\pi = 0$

b $2x + 2\sqrt{2}y + 4 - \pi = 0$

c $6x + 12y - 6\sqrt{3} - \pi = 0$

d $12\sqrt{3}x - 24y + 12 - \sqrt{3}\pi = 0$

e $12x - 2y + 2 - \pi = 0$

7 **a** $v = -\frac{3}{2}\sin\left(\frac{t}{2}\right)$ **b** $a = -\frac{3}{4}\cos\left(\frac{t}{2}\right)$

c $\pi, 3\pi, 5\pi, \ldots$ s **d** $\pm\frac{3}{2}$ cm/s; 0 cm/s^2

e 0.75 at $2\pi, 6\pi, \ldots$ s

8 **a** $a = 2.52\pi\cos(2\pi t)$ **b** 0 m/s; 7.9 m/s^2

c $\frac{3}{4}, \frac{7}{4}, \frac{11}{4}, \ldots$s

9 Approximately 18 cm/min.

10 **a** $x = 2\sin(3t)$

$v = \frac{dx}{dt} = 6\cos(3t)$

$a = \frac{dv}{dt} = -18\sin(3t)$

$= -9[2\sin(3t)]$

$= -9x$

b $x = a\cos(nt)$

$v = \frac{dx}{dt} = -an\sin(nt)$

$a = \frac{dv}{dt} = -an^2\cos(nt)$

$= -n^2[a\cos(nt)]$

$= -n^2x$

CHAPTER 1 REVIEW

1 B

2 B

3 D

4 C

5 E

6 D

7 B

8 **a** $5x^4(3x^2 + 2x - 5) + 2x^5(3x + 1)$

b $2x(x^3 - 4x - 1) + (x^2 + 1)(3x^2 - 4)$

9 **a** $\frac{5x^2(4x+3)}{(2x+1)^2}$ **b** $\frac{4x^2 - 6x + 5}{(4x-3)^2}$

10 **a** $-6x^{-7}$ **b** $\frac{1}{7}x^{-\frac{6}{7}}$ **c** $\frac{3\sqrt{x}}{2}$ **d** $-\frac{18}{x^3}$

11 **a** $10(2x-7)^4$

b $24x(3x+1)(2x^3 + x^2 - 3)^3$

12 **a** $-15x^4(x^5+1)^{-4}$ **b** $\frac{1}{3\sqrt[3]{(x-1)^2}}$

13 **a** $6x(x+2)^8 + 24x^2(x+2)^7 = 6x(x+2)^7(5x+2)$

b $\frac{-6x(5x+1)}{(5x-1)^5}$

14 **a** e^3 **b** $3x - e^2y + 9 = 0$

15 **a** $2x + 2e^x$ **b** $6e^{6x}$ **c** $9e^x(e^x - 3)^8$

d $8e^{4x+1}$ **e** $e^x - e^{-x}$

16 **a** $4e^{2x} + 2e^{2x}(4x+3) = 2e^{2x}(4x+5)$

b $\frac{e^{3x}(3x-13)}{(x-4)^2}$

17 **a** 1000 apps

b **i** 4221 apps **ii** 317 348 apps

c **i** 1013 apps/month **ii** 76 164 apps/month

18 **a** **i** 55.6° **ii** 35.4°

b **i** 8.3°/min **ii** 5.3°/min

19 **a** $Q = 375e^{0.04t}$ **b** 476.7 g

c 19.1 g/week

20 **a** $3\cos(x)$ **b** $5\cos(5x)$

c $\sin(2x) + 2x\cos(2x)$

d $7[3 - \cos(x)][3x - \sin(x)]^6$

e $3x^2\cos(x^3 + 1)$

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21 a $-\frac{1}{3}\sin\left(\frac{x}{3}\right)$

b $e^x[\cos(x) - \sin(x)]$

c $-15\sin(3x)[2+\cos(3x)]^4$

d $-\pi\sin(\pi x)$

e $-2\cos(x)\sin(x)$

22 a $\frac{6}{\cos^2(x)}$ b $\frac{3}{\cos^2(3x)}$ c $\frac{\pi}{5\cos^2\left(\frac{\pi x}{5}\right)}$

d $3x^2\tan(2x) + \frac{2x^3}{\cos^2(2x)}$ e $\frac{x - \tan(x)\cos^2(x)}{x^2\cos^2(x)}$

23 Proof

24 a $4x - 2y - \pi + 2 = 0$

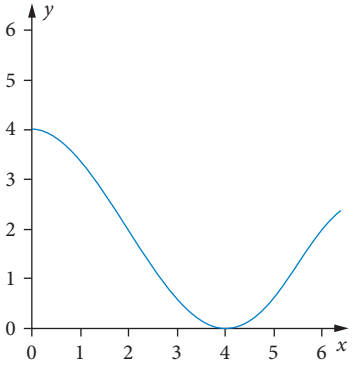
b $\left(\frac{\pi-2}{4}, 0\right); \left(0, \frac{2-\pi}{2}\right)$

c $\frac{(\pi-2)^2}{16}$ units^2

25 a −5.8 cm b 0, π, 2π, … s

c −6 cm/s

26 a



b About 57.5°.

2.01

1 a Continuous b Discrete

c Discrete d Continuous

e Discrete f Continuous

g Discrete h Continuous

2 a $X = \{1, 2, 3, 4, 5, 6\}$

b $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

c $X = \{0, 1, 2, 3\}$

d $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

e X = any real number between 0 and 2.

3 a Discrete, random

b Discrete, random

c Discrete, random

d Discrete, non-random (the number chosen by a person could be influenced by their personal preferences and is therefore not random).

e Continuous, random

4 D

5 C

6 B

7 E

8 D

9 D

10 B

11 a 0.981 b $\frac{6}{19}$

12 $(2, \frac{1}{16}), (3, \frac{1}{8}), (4, \frac{3}{16}), (5, \frac{1}{4}), (6, \frac{3}{16}), (7, \frac{1}{8}), (8, \frac{1}{16})$

13 a 0.125 b 0.0625 c 0.8125

d 0 e 0.5

f $\{0, 1, 2, 3, 4\}$, $p(0) = 0.0625$, $p(1) = 0.4375$, $p(2) = 0.3125$, $p(3) = 0.125$, $p(4) = 0.0625$

14 $M = 1, 2, 3, 4, 5, 6$; $P(M = m) =$ $(1, \frac{11}{36}), (2, \frac{1}{4}), (3, \frac{7}{36}), (4, \frac{5}{36}), (5, \frac{1}{12}), (6, \frac{1}{36})$

15 a $T = 0, 1, 2, 3, 4$;

$P(T = t) = (0, \frac{1}{16}), (1, \frac{1}{4}), (2, \frac{3}{8}), (3, \frac{1}{4}), (4, \frac{1}{16})$

b $\frac{5}{16}$

2.02

1 A

2 D

3 B

4 E

5 B

6 B

7 a No, as $p(x)$ does not add to 1.

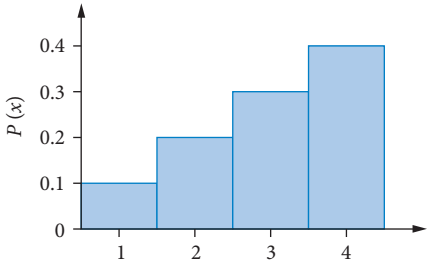
b Yes

c No, as $p(t)$ has a negative value.

8 a

x	$p(x)$
1	0.1
2	0.2
3	0.3
4	0.4

b



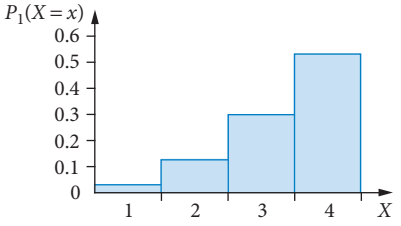
c $0 \le P(X = x) \le 1$ for all values of x; $\sum P(X = x) = 1$ and every value of x has a unique value of $p(x)$.

9 a

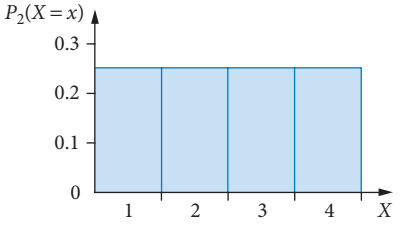
x	1	2	3	4
$P_1(X = x)$	$\frac{1}{30}$	$\frac{4}{30}$	$\frac{9}{30}$	$\frac{16}{30}$
$P_2(X = x)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

b For both distributions, $0 \leq p(x) \leq 1$ and $\sum p(x) = 1$.

c $P_1(X = x)$



$P_2(X = x)$



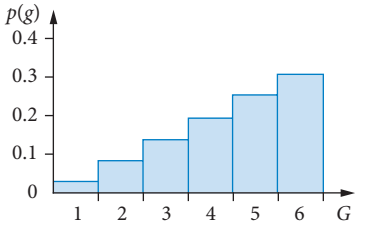
10

f	2	3	4	5	6	7	8
$P(F=f)$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

11 a

g	1	2	3	4	5	6
$P(G=g)$	$\frac{1}{36}$	$\frac{1}{12}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{1}{4}$	$\frac{11}{36}$

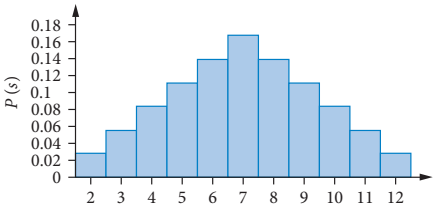
b



12 a

x	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

b

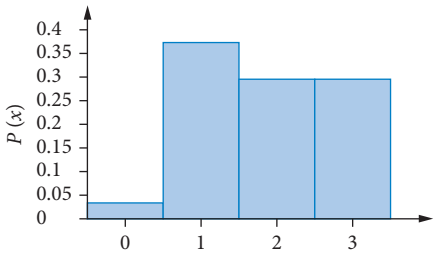


c $0 \leq P(X = x) \leq 1$ for all values of x; $\sum P(X = x) = 1$ and every value of x has a unique value of $p(x)$.

13 a

x	0	1	2	3
$p(x)$	$\frac{1}{27}$	$\frac{10}{27}$	$\frac{8}{27}$	$\frac{8}{27}$

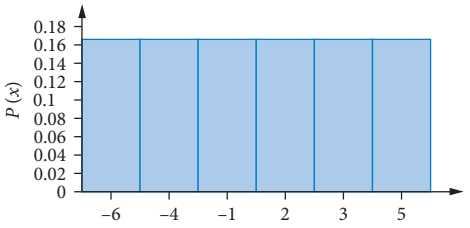
b



14 a

x	−6	−4	−1	2	3	5
$p(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

b



15

x	0	1	2	3	4
$p(x)$	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

16

x	−5	5	20
$p(x)$	$\frac{5}{9}$	$\frac{5}{12}$	$\frac{1}{36}$

2.03

1 0.289

2 **a** 0.178 **b** 0.185

3 0.4

4 **a** 0.279 **b** 0.097

5 **a** 0.322 **b** 0.107

INVESTIGATION: RANDOM NUMBERS

The more results that are combined, the closer the graph comes to a uniform distribution.

2.04

1 A

2 B

3 **a** $N = 1, 2, 3, 4$; uniform

b $X = 0, 1, 3, \ldots, 9$; uniform

c $S = 0, 1, 3, \ldots, 9$; uniform

d $D = 1, 2, 3, \ldots, 20$; non-uniform.

e $M = 1, 2, 3, 4, 5, 6$; uniform

f $T = 2, 3, 4, \ldots, 12$; non-uniform

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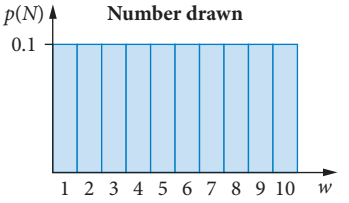
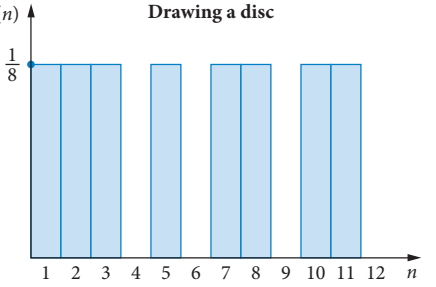
4



The probability distribution for N is uniform.

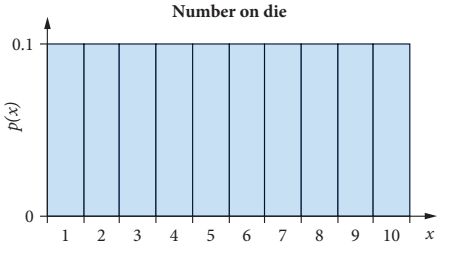
5 a



b $\frac{3}{8}$ **c** $\frac{7}{8}$ **d** $\frac{3}{8}$

e $\frac{5}{8}$ **f** $\frac{5}{8}$

6 a



b 0.9 **c** 0.3 **d** 0.4

7 a $\frac{1}{30}$ **b** $\frac{29}{30}$ **c** $\frac{3}{5}$

d $\frac{13}{30}$ **e** $\frac{17}{30}$

8 a $\frac{240}{571} \approx 0.4203$ **b** $\frac{150}{571} \approx 0.2627$

c $\frac{241}{571} \approx 0.4221$ **d** $\frac{301}{571} \approx 0.5271$

2.05

1 A
2 D
3 B
4 B
5 D

6 a X = number of hearts selected; $N = 52$; $n = 4$; $k = 13$; $X = 0, 1, 2, 3, 4$

b X = number of faulty transistors selected; $N = 6$; $n = 3$; $k = 2$; $X = 0, 1, 2$

c X = number of defective chips selected; $N = 50$; $n = 10$; $k = 9$; $X = 0, 1, 2, \ldots, 9$

d X = number of defective boards selected; $N = 200$; $n = 40$; $k = 20$; $X = 0, 1, 2, \ldots, 20$

e X = number of workstations requiring a special login code selected; $N = 95$; $n = 12$; $k = 35$; $X = 0, 1, 2, \ldots, 12$

7 a 0.2135 **b** 0.6 **c** 0.0552

d 0.0012 **e** 0.1770

8 a 0.1136 **b** 0.2979 **c** 0.1523

d 0.0908 **e** 0.0217

9 0.3151

10 a 0.2705 **b** 0.5777

11 a 0.0833 **b** 0.7282

12 a 0.000 000 1228 **b** 0.0238

13 a 0.3590 **b** 0.0507 **c** 0.0098

14 0.5696

2.06

1 C
2 D
3 B
4 5.5

5 a 4 **b** −1 **c** 3

6 2.8

7 a $E(F) = 57.5$ **b** $E(M) = 17.25$

8 a About 1437 **b** About 315

9 $2\frac{7}{12}$

10 a 1.375 **b** 1.852

11 a

s	1	2	3	4	5	6
$p(s)$	$\frac{11}{36}$	$\frac{1}{4}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{1}{12}$	$\frac{1}{36}$

b 2.528

12 a

s	3	4	5	6	7	8	9
$P(S = s)$	0.1	0.1	0.2	0.2	0.2	0.1	0.1

b $E(S) = 6$

13 2

14 a $k = 0.05$

b The value of X is more likely to be 1 or 2 than it is to be 4 or 5. This means that $E(X)$ is likely to be less than 3.

c 2.15

15 $x = \frac{1}{12}$ and $y = \frac{5}{24}$.

16 1.9

17 1

2.07

1 A
2 A
3 A
4 $Var(W) \approx 3.24$, $SD(W) \approx 1.8$
5 84

6 7.62

7 a

x	3	4	6	7	8	9
$p(x)$	0.05	0.05	0.15	0.45	0.2	0.1

b 6.9 c About 1.99.

8 a

n	0	1	2	3	4	5	6	7	8	9
$p(n)$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{1}{15}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{2}{15}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{15}$

b 5 c About 5.07.

d About 2.25.

9 a $\mu = 1.3$; $Var(N) \approx 2.01$; $\sigma \approx 1.42$

b $\mu = 13.54$; $Var(R) \approx 2.67$; $\sigma \approx 1.63$

10 a

Z	0	1	2	3
$P(Z = z)$	$\frac{1}{11}$	$\frac{3}{22}$	$\frac{3}{11}$	$\frac{1}{2}$

b 2.182 c 0.983

11 About 6.05.

12 a $\mu \approx 1437$; $Var(X) \approx 13\,158$; $\sigma \approx 114.7$

b $\mu \approx 315$; $Var(Y) \approx 195$; $\sigma \approx 14$

13 a 59.42 b 16.1 c 4.01

14 $k = 5$

15 $k = 3$

16 a 7 b $5\frac{5}{6}$ c About 2.42.

17 a

t	4	6	8	10	12
$p(t)$	$\frac{1}{36}$	$\frac{1}{9}$	$\frac{5}{18}$	$\frac{1}{3}$	$\frac{1}{4}$

b $E(T) = 9\frac{1}{3}$ and $Var(T) = 4\frac{4}{9}$

18 a

x	2	3	4	5	6
$p(x)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{9}$

y	1	2	3
$p(y)$	$\frac{5}{9}$	$\frac{1}{3}$	$\frac{1}{9}$

b 4, 1.33 c 1.56, 0.469

INVESTIGATION: LIFE INSURANCE

The actual life insurance cost is higher than the theoretical cost based on the probability of death. The differences are because of the costs of administration and the profits made by the companies.

2.08

1 a −$2.64 b No

2 8%

3 $14

4 2.65

5 a 0.0775 b 0

6 a About 0.763 b About 2.513

c About 1.4

7 The player can expect to win $0.80 playing this game, so it favours the player.

8 a The player can now expect to lose $0.20 playing this game, so it favours the house.

b The player can now expect to lose $0.40 playing this game, so it is even more favourable to the house.

9 No, you are likely to lose about $7 on average for each ticket purchased.

10 You can expect to lose about 22c on each play.

11 $1225

12 $675

13 a $25 b $3000

14 a 0.178 b $335.60 c $300

15 a $0.33 b 33c or $1 for 3 games

16 a $2400 b −$1000

17 2.875

18 You would expect the standard deviation for the first exam to be more than in the second.

19 $270 000

CHAPTER 2 REVIEW

1 D

2 E

3 E

4 A

5 B

6 A

7 A

8 C

9 B

10 B

11 C

12 a 0.06 b 0.49 c About 0.089

13 a Discrete b Continuous

c Discrete d Continuous

e Discrete f Discrete

g Continuous

h Discrete (must be to 5 cents)

14 a Yes b Yes

c No, as $p(x)$ has a negative value.

15 a

x	1	2	3	4
$P_1(X = x)$	0.1	0.2	0.3	0.4
$P_2(X = x)$	0.25	0.25	0.25	0.25

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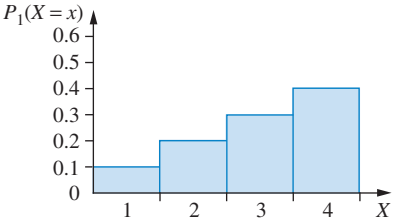
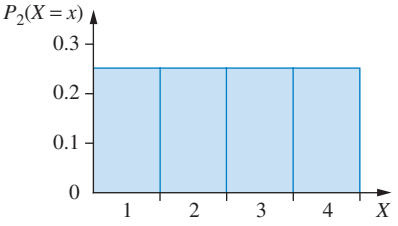
b





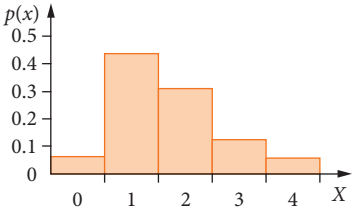
c $\sum p(x) = 1$ and $0 \le p(x) \le 1$, for each distribution.

d The second distribution is uniform.

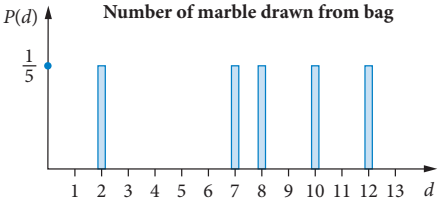
16 a

x	0	1	2	3	4
$P(X = x)$	$\frac{1}{16}$	$\frac{7}{16}$	$\frac{5}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

b



17 a



b $P(d \text{ is odd}) = \frac{1}{5}$

c The distribution is uniform.

18 a

t	11	12	13	14	15	16	17
f	2	3	4	6	1	3	0
$P(T = t)$	0.08	0.12	0.16	0.24	0.04	0.12	0

t	18	19	20
f	1	1	4
$P(T = t)$	0.04	0.04	0.16

b 0.12 **c** 0.4

19 a $N = 40$, $n = 5$, $k = 7$, $X = 0, 1, 2, \ldots, 5$

b $N = 5000$, $n = 10$, $k = 1000$, $X = 0, 1, 2, \ldots, 10$

c $N = 100$, $n = 15$, $k = 12$, $X = 0, 1, 2, \ldots, 12$

20 a 0.0918 **b** 0.2415 **c** 0.000 656

21 a

d	0	1	2	3	4	5
$P(D = d)$	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$

b $E(D) = 1\frac{17}{18} \approx 1.94$

22 $E(T) \approx 5.27$; $Var(T) \approx 5.83$

23 $E(X) = 1$; $Var(X) = 4$

24 a 450 **b** 17 500

c About 132.3

25 a

x	0	1	2	3
$P(X = x)$	$\frac{1}{64}$	$\frac{9}{32}$	$\frac{9}{32}$	$\frac{27}{64}$

b $\frac{135}{64} \approx 2.1$

26 $E(X) = 12.25$; $Var(X) \approx 80$; the distribution is non-uniform.

27 a 0.1620 **b** 0.5158

28 a 0.125 0028 **b** \$0.53 **c** 47%

29 a \$1.67

b No, you should win in the long run.

c Change the sums that are included so winning and losing were equal. Eg. less than 6 or an 8, 11 or 12; more than 7 or a 4.

30 a \$2400 **b** −\$1000

31 The expected value is about −23c, so the player loses 23c.

32 \$4980

3.01

1 $9\frac{2}{9} = 9.\overline{2}$

2 $5\frac{1}{15} = 5.0\overline{6}$

3 a $7\frac{1}{14} \approx 7.071$ **b** $3\frac{1}{27} \approx 3.037$

c $50\frac{9}{25} = 50.36$ **d** $3\frac{7}{405} \approx 3.017$

e $2\frac{1}{32} \approx 2.031$

4 16, 16.5

5 10 420 000

6 268.8 compared with 269.042 006 25 (exact)

7 \$316

8 6%

9 a $6x^2 - 6x + 4$ **b** 40 **c** 0.8

10 1.35

11 $28.8\pi \text{ cm}^3 \approx 90.48 \text{ cm}^3$

12 a 13.26 cm **b** $\delta V = 9\pi\delta x$ mL

c $\delta V = 125\delta y$ mL **d** About 2.8 mL.

e About 25 mL.

13 a 9.72 m/s^2 **b** $\delta g \approx -0.34\delta t$

c 0.17 m/s^2

14 2000 mm^2

15 a \$14 400 **b** \$23 040 **c** \$28 800

16 6%

17 1%

3.02

1 $7x^6 - 10x^4 + 4x^3 - 1$; $42x^5 - 40x^3 + 12x^2$

2 $72x^7$

3 $f'(x) = 10x^4 - 3x^2, f''(x) = 40x^3 - 6x$

4 $42x^5 - 40x^3 + 48x^2$

5 $-20\cos(2x)$

6 a $\frac{dy}{dx} = 4x - 3, \frac{d^2y}{dx^2} = 4$ b $-4x^{-5}, 20x^{-6}$

7 $f'(1) = 11, f''(-2) = 168$

8 $f'(-1) = -16, f''(2) = 40$

9 $g''(4) = -\frac{1}{32}$

10 $\frac{d^2h}{dt^2} = 26$ when $t = 1$

11 $f'(x) = -\frac{1}{2\sqrt{2-x}}; f''(x) = -\frac{1}{4\sqrt{(2-x)^3}}$

12 $f'(x) = -\frac{16}{(3x-1)^2}; f''(x) = \frac{96}{(3x-1)^3}$

13 $\frac{d^2v}{dt^2} = 24t + 16$

14 $x = \frac{7}{18}$

15 $x > \frac{1}{6}$

16 $\frac{dy}{dx} = 20\cos(x)\left[4\sin(x) - 2\right]^4;$

$\frac{d^2y}{dx^2} = 320\cos^2(x)\left[4\sin(x) - 2\right]^3$

$-20\sin(x)\left[4\sin(x) - 2\right]^4$

17 $f'(x) = \cos\left(\frac{x}{2}\right) + 3\sin(2x);$

$f''(x) = -\frac{1}{2}\sin\left(\frac{x}{2}\right) + 6\cos(2x)$

18 $b = \frac{2}{3}$

19 $b = -2.7$

20 velocity = 20 m/s; acceleration = 18 m/s^2

21 a -4 m/s b 8 m/s
c 0 m/s^2 d 18 m/s^2

22 a 11 m/s b -9 m/s
c 2 m/s^2 d -22 m/s^2

23 a $v = 3t^2 + 12t - 2; a = 6t + 12$
b 266 m c 133 m/s d 42 m/s^2

24 a $v = 2 - 10t$ b -98 m/s
c $a = -10 = g$

25 $v = \frac{17}{(3t+1)^2}; a = -\frac{102}{(3t+1)^3}$

3.03

1 $x > -\frac{1}{3}$

2 $x < 3$

3 $\frac{d^2y}{dx^2} = -8 < 0$

4 $\frac{d^2y}{dx^2} = 2 > 0$

5 $x < 2\frac{1}{3}$

6 $-2 < x < 1$

7 a No
b Yes—inflection at (0, 0).
c Yes—inflection at (0, 0).
d Yes—inflection at (0, 0).
e No

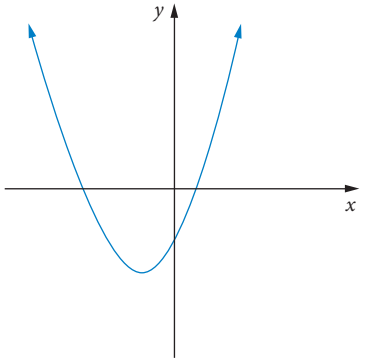
8 (1, 9)

9 (1, −17) and (−1, −41)

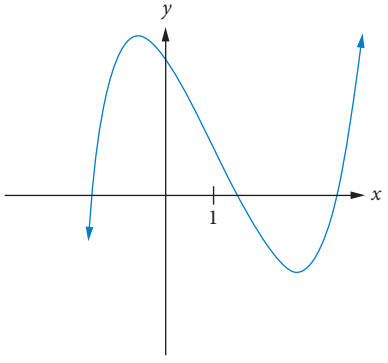
10 None: (2, 31) is not an inflection since concavity does not change.

11 (0, 7), (1, 0) and (−1, 14)

12 Other answers are possible.



13 Other answers are possible.



14 $f''(x) = \frac{12}{x^4}$

$x^4 > 0$ for all $x \neq 0$

So $\frac{12}{x^4} > 0$ for all $x \neq 0$

So the function is concave upward for all $x \neq 0$.

15 a $\frac{d^2y}{dx^2} = 12x^2 + 24$

$x^2 \geq 0$ for all x

So $12x^2 \geq 0$ for all x

$12x^2 + 24 \geq 24$

So $12x^2 + 24 \neq 0$ and there are no points of inflection.

b $12x^2 + 24 \geq 24$ The curve is always concave upwards.

16 $a = 2$

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17 $p = 4$

18 $a = 3, b = -3$

3.04

1 (1, 0) minimum

2 (0, 1) minimum

3 (2, −5), $\frac{d^2y}{dx^2} = 6 > 0$

4 (0.5, 0.25), $\frac{d^2y}{dx^2} < 0$, so a maximum

5 (0, −5), horizontal point of inflection

6 Yes—horizontal point of inflection at (0, 8).

7 (−2, −78) minimum, (−3, −77) maximum

8 (0, 1) maximum, (−1, −4) minimum, (2, −31) minimum

9 (0, 1) maximum, (0.5, 0) minimum, (−0.5, 0) minimum

10 (4, 176) maximum, (5, 175) minimum

11 (3.67, 0.38) maximum

12 (0, −1) minimum, (−2, 15) maximum, (−4, −1) minimum

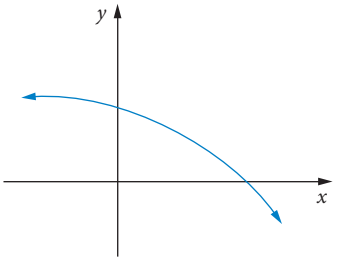
13 a $a = -\frac{2}{3}$ b maximum, as $\frac{d^2y}{dx^2} < 0$

14 $m = -5\frac{1}{2}$

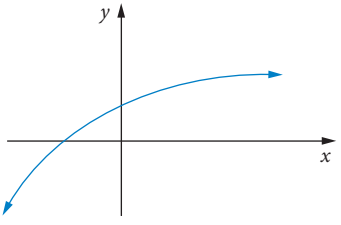
15 $a = 3, b = -9$

3.05

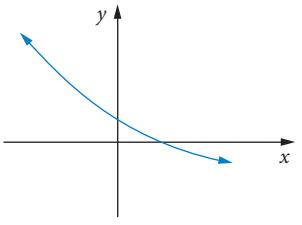
1 a



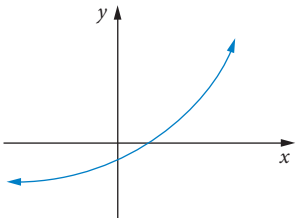
b



c



d



2 a $\frac{dy}{dx} > 0, \frac{d^2y}{dx^2} > 0$

b $\frac{dy}{dx} < 0, \frac{d^2y}{dx^2} < 0$

c $\frac{dy}{dx} > 0, \frac{d^2y}{dx^2} < 0$

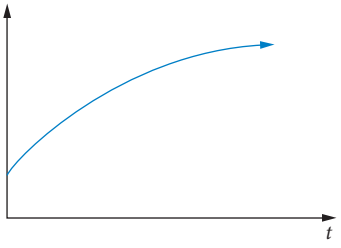
d $\frac{dy}{dx} < 0, \frac{d^2y}{dx^2} > 0$

e $\frac{dy}{dx} > 0, \frac{d^2y}{dx^2} > 0$

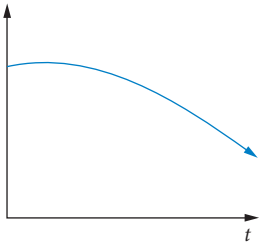
3 a $\frac{dP}{dt} > 0, \frac{d^2P}{dt^2} < 0$

b The rate is decreasing.

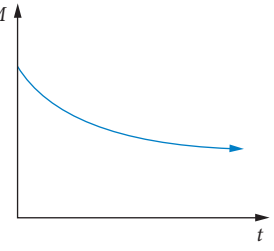
4 I



5 S



6 M

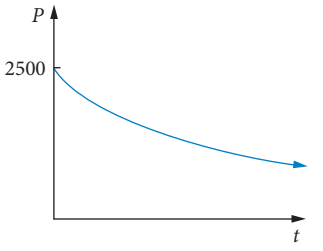


7 $\frac{dM}{dt} < 0, \frac{d^2M}{dt^2} > 0$. The mass is decreasing but at a decreasing rate.

8 a The number of fish is decreasing.

b The population rate is increasing.

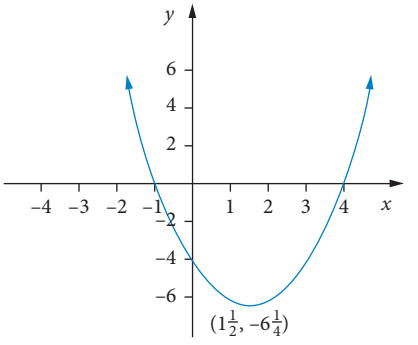
c



9 The level of education is increasing, but the rate is slowing down.

10 The population is decreasing, and the population rate is decreasing.

11



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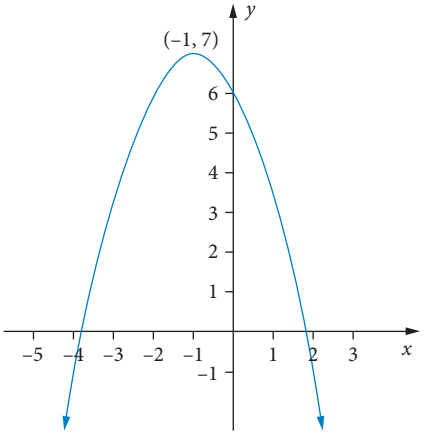

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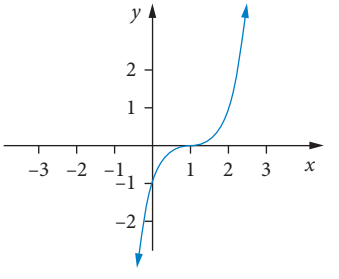

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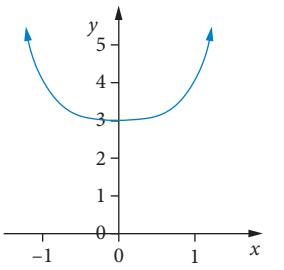

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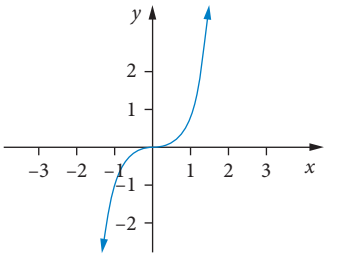

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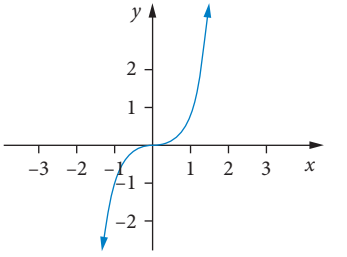

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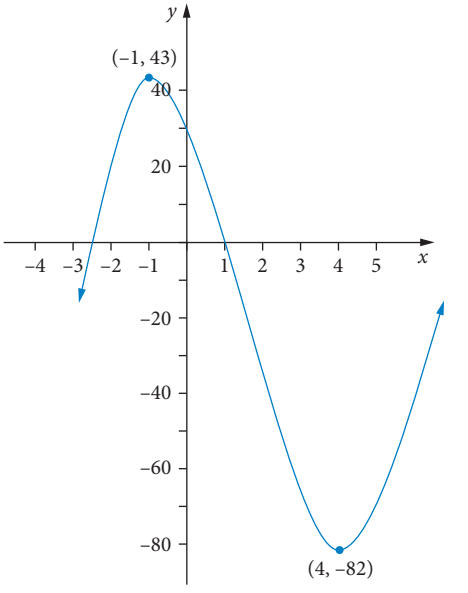
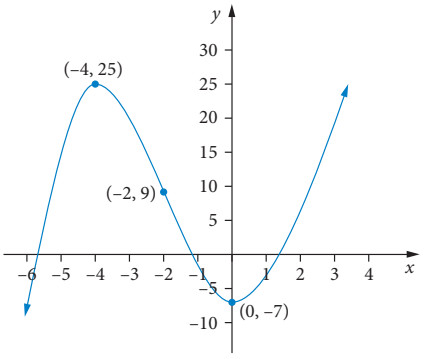


9780170250290

18 **a** (0, −7) minimum, (−4, 25) maximum

b (−2, 9)

c



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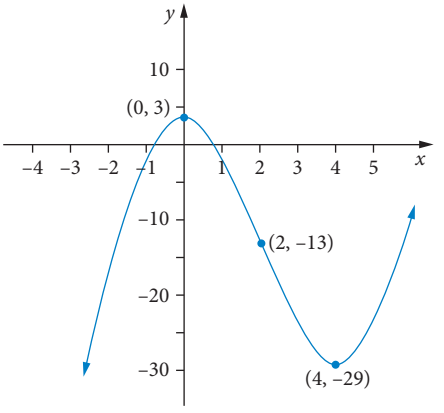

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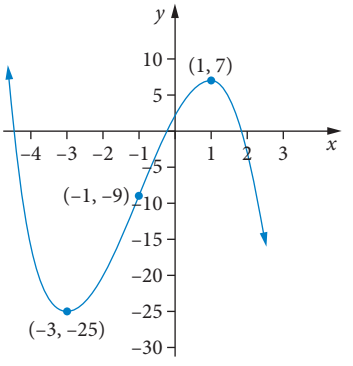

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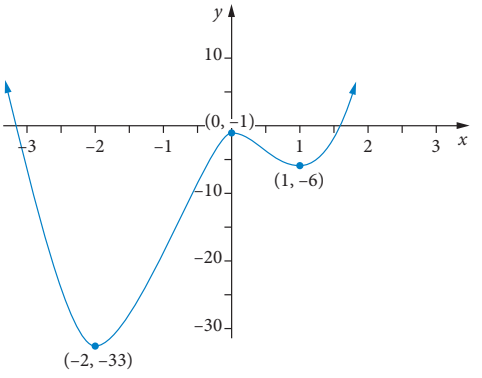

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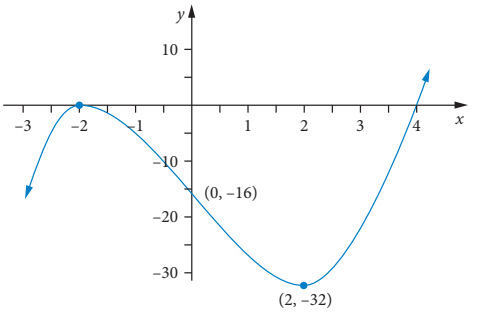

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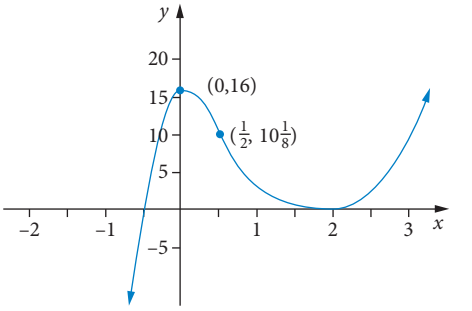
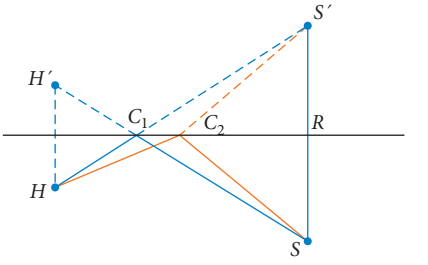


INVESTIGATION: HERON'S PROBLEM

Consider the case where the house is closer to the bank than the shed. If it's the other way round, just rename *H* and *S* the other way round.



Put a sticks at *S*′ and *H*′ on the other side of the river, directly opposite the shed and house and the same distance from the riverbank. Set off towards the stick. When you get to the riverbank, get the water and go directly to the shed.

The path HC_1S' is the same length as HC_1S as $\triangle C_1SR$ is congruent to $\triangle C_1S'R$.

The same applies to any other path like as HC_2S' and as HC_1S.

The shortest path between *H* and *S*′ is a straight line, so the path described is the shortest.

3.06

1 $a = 6, b = 4.5$

2 Both 12.5.

3 20 and −20

4 Both 16.

5 (3.5, 3.5)

6 $\left(3\frac{1}{2}, \sqrt{\frac{7}{2}}\right)$

7 a $y = \frac{3}{4}x + 2\frac{1}{4}$ b $y = -\frac{3}{4}x + 2\frac{1}{4}$

8 10

9 About 1.18 km

10 20 km/h

11 About 15 m

12 50 km/h

13 About 63.9 m

14 a $\frac{1}{20}$ b 0.36 months (10 or 11 days)

15 About 71° (70.528…), taking about 10.7 min to finish.

3.07

1 500 000 m^2

2 $1 : 2\sqrt{2} \approx 1 : 2.83$

3 Width ≈ 12 cm, depth ≈ 6 cm

4 10 cm × 10 cm × 5 cm

5 54 m^2

6 5.2 cm

7 2 cm × 2 cm

8 About 7.79 cm

9 Height ≈ 1.68 m, width ≈ 3.36 m

10 Radius ≈ 1.89 m $\left(\frac{4\sqrt{2}}{3}\text{ m}\right)$, height ≈ 2.67 m $\left(2\frac{2}{3}\text{ m}\right)$

11 $\frac{18}{4+\pi}\ m^2$

12 4000 cm^3

13 Height ≈ 50 cm, diameter ≈ 31 .8 cm

14 6 m × 6 m × 9 m

15 a About 2.62 cm × 8.10 cm × 14.77 cm.
b About 3.92 cm × 12.15 cm × 22.15 cm.
c About 6.54 cm × 20.25 cm × 36.92 cm.

INVESTIGATION: FUN RIDES

1 He should drop the price by $3.75 to $6.25.

2 The amount extra is $262.50.

3 Changing parameters (the charges, proportion full, capacity of the fun ride) will change the point of highest return.

3.08

1 15 t per month

2 11

3 a $P(x) = 2.7x - 0.001x^2 - 50$ b 1350

4 a 2500 b 2000

5 a 1500 b 3500, $8250

6 $162

7 a 1500 b 3500, $82 500

8 a $C(x) = 100 + 250x - 5x^2 + \frac{x^3}{3}$
b $R(x) = x(5000 - 5x)$
c $P(x) = -100 + 4750x - \frac{x^3}{3}$
d 69

9 7 (integer value)

10 75.8 km/h, $53.58

3.09

1 84 m

2 4.25 cm

3 18.75 m

4 11 m

5 40 m

6 38.2 m

7 Minimum of about 217 at 2°C, maximum of about 1189 at 11°C.

8 55 trees/ha

9 About 11.34 km along the coast (i.e 1.66 km from the marina).

10 He should walk through the bush to a point 3.75 km down the main road from the nearest point (i.e. 1.25 km from the service station) and then walk down the road to get there in 2 h 20 m.

CHAPTER 3 REVIEW

1 C

2 B

3 A

4 C

5 D

6 a 4.05 b 3.985

7 about 18.1 cm^3

8 a diameter ≈ 36.3 cm, height ≈ 58.0 cm
b x% c $2y$% d 9%

9 a $30x^5 - 6x + 1$, $150x^4 - 6$
b $8(2t + 9)^3$, $48(2t + 9)^2$
c $54n^2 + 48n - 22$, $108n + 48$
d $\frac{21}{(3x-1)^2}, -\frac{126}{(3x-1)^3}$

10 a $\frac{dx}{dt} = 3t^2 - 24t + 36$ b $\frac{d^2x}{dt^2} = 6t - 24$
c $-12\ cm/s^2$

11 $x > 1\frac{1}{6}$

12 Yes, point of inflection at (0, 0).

13 (−4, −767) and (−2, −143)

14 a Minimum at (1, 0).
b Maximum at (2.5, 6.25).
c Maximum at (0, 7) and minimums at (−1, 2) and (2, −25).
d Minimum at (0.5, 0).

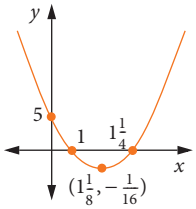
15 Point of horizontal inflection at (0, −1).

16 a $\frac{dP}{dt} > 0$ and $\frac{d^2P}{dt^2} < 0$

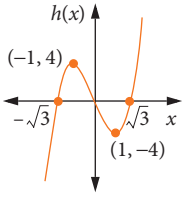
b The number of possums is increasing.

c The rate of growth of the population is decreasing.

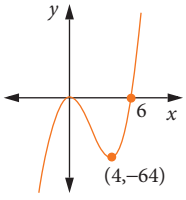
17 $y = 4x^2 - 9x + 5$



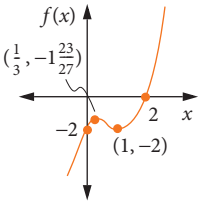
$h(x) = 2x^3 - 6x$



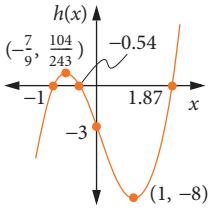
$y = 2x^3 - 12x^2$



$f(x) = x^3 - 2x^2 + x - 2$



$h(x) = 3x^3 - x^2 - 7x - 3$



18 $x = 4, y = 12$

19 2 trees

20 a 8:24 a.m.

b 6 km.

21 160 m by 320 m

22 16 cm by 8 cm

23 9 600 cm^2

24 5

25 a 299 b \$150.50 c \$44 900.50

26 4 m

27 35°C

MIXED REVISION 1

Multiple choice

1 E

2 D

3 E

4 C

5 C

6 C

7 C

8 B

9 B

Short answer

1 a 175 b 7

2 a $(2x + 5)(8x^7 - 3) + 2(x^8 - 3x + 4)$

b $7(2x + 3)(x^2 + 3x + 4)^6$

c $\frac{31}{(3x + 7)^2}$ d $2e^{2x} + \frac{1}{\cos^2(x)}$

e $\frac{1}{4}\cos\left(\frac{x}{4}\right)$

3 26 m

4 a

X	2	3	4	5
$P(X = x)$	$\frac{5}{58}$	$\frac{5}{29}$	$\frac{17}{58}$	$\frac{13}{29}$

b About 4.103. c About 0.955.

d About 91.4%. 5 60 cm by 60 cm by 15 cm.

6 a $12x - 3y + 3\sqrt{3} - 4\pi = 0$

b $x - 4y + 5 = 0$

Application

1 a

R	0	1	2	3
$P(R = r)$	0.064	0.288	0.432	0.216

b $\frac{6}{13}$

2 a $32e^{20}$ cm/s^2

b $x = 2e^{4t}$

$v = 8e^{4t}$

$a = 32e^{4t}$

$= 16(2e^{4t})$

$= 16x$

3 \$2 327.09

4 No, the game is not fair as you stand to lose 10c every time you play.

5 50

6 a 241 650 b 16 915 people/year

c 12 165 people/year

4.01

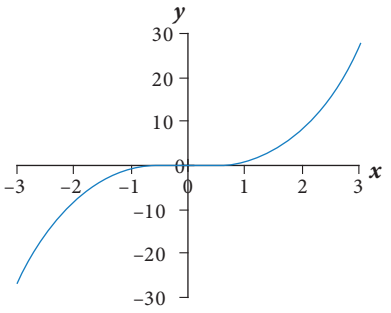
1 a 320 km b 245 km c 250 km
2 a 6 units^2 b 21 units^2
3 a 50 units^2 b 96 units^2 c 100 units^2
d 8 units^2 e 36 units^2
4 480 km
5 a 18 units^2 b 8750 units^2 c 360 units^2
d 90 units^2 e 600 units^2
6 8 units^2
7 a 180 units^2 b 210 units^2
8 a $6.25\pi \approx 19.63$ units^2
b i 20.25 units^2 ii 15.125 units^2
9 a 2100 kL b 1750 kL
10 a 14.14 units^2
b i 18 units^2 ii 9 units^2 iii 13.5 units^2

4.02

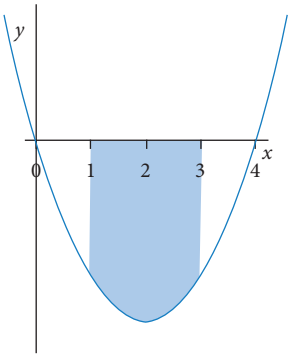
1 a 3.625 units^2 b 286.125 units^2
c 100 units^2 d 0.44 units^2
e 10 units^2
2 a 20.95 units^2 b 1.5 units^2
c 9.33 units^2 d 7.64 units^2
e 1.6 units^2
3 a 1.24 units^2 b 1.039 units^2
4 a i 1.625 units^2 ii 3.125 units^2
iii 2.259 units^2
b i 26 units^2 ii 50 units^2
iii 34.96 units^2
c i 3 units^2 ii 11 units^2 iii 5.61 units^2
d i 6.25 units^2 ii 11.25 units^2 iii 8.17 units^2
e i 85.79 units^2 ii 233.2 units^2 iii 129.75 units^2
5 a 2.33 units^2 b 0.245 units^2
c 11.25 units^2 d 65.25 units^2
e 0.46 units^2
6 a 4.5 units^2 b 4.5 units^2
7 5 units^2
8 a i 0.298 units^2 ii 0.278 units^2
iii 0.289 units^2
b 0.292 units^2 c 0.288 units^2
9 a 3.21 km^2 b 2.7285 km^3

4.03

1 a 10.625 units^2 b 194.06 units^2
c 5.69 units^2 d 979 units^2
e 1.98 units^2
2 a 4.33 b 395.63 c 155.25
d 9.75 e 402
3 a 20.95 b 0.85 c 6.14
d 2.44 e 453
4 $\frac{\pi(2+\sqrt{2})}{8}$
5 a 35.28 b 25.52
6 5.402
7 68.425
8 a 3.968 b 0
c



The two areas cancel each other out.
9 −7.33; negative since it is below the x-axis as the graph shows.



10 a 26.75 m b $26\frac{2}{3}$ m

INVESTIGATION: AREAS UNDER A CURVE

1 $y = x$
2 $y = 3x$
3 $y = 4x$
4 $y = 0.5x^2$
5 $y = x^2$
6 $y = 1.5x^2$

The original function is the derivative of the area function.

4.04

1 a i 36 ii 189 iii 225
b $\int_1^4 x^3\,dx + \int_4^6 x^3\,dx = 36 + 189 = 225 = \int_1^6 x^3\,dx$
2 a i 8.58 ii 24.43 iii 33.01
b $\int_0^2 (x^2+3)dx + \int_2^4 (x^2+3)dx = \int_0^4 (x^2+3)dx$
3 a $\int_0^5 x^2\,dx$ b $\int_1^7 (x+1)dx$
c $\int_{-2}^2 (x^3 - x - 1)dx$ d $\int_0^3 (2x+1)dx$
e $\int_1^3 6x^3\,dx$ f $\int_{-1}^3 (3x^2 - 4x - 1)dx$

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g $\int_{-2}^{2}(x^2-2)dx$ h $\int_{0}^{7}3\,dx$

i $\int_{1}^{3}5x^4dx$ j $\int_{0}^{6}(2x-3)dx$

4 a i 332.5 ii 997.5

b $3\int_{0}^{10}x^2dx = 3\times166.25 = 498.75 = \int_{0}^{10}3x^2dx$

5 a i 2593.5 ii 5187

b $2\int_{2}^{5}x^5dx = 2\times2593.5 = 5187 = \int_{2}^{5}2x^5dx$

6 a i 4.125 ii 3.9375 iii 8.0625

b $\int_{1}^{2}2x^2dx + \int_{1}^{2}3xdx = 4.125 + 3.9375 = 8.0625$

$= \int_{1}^{2}(2x^2+3\boldsymbol{x})dx$

7 a $\int_{0}^{2}(3x^2+2x+2)dx$ b $\int_{1}^{2}(3x^3-3x+1)dx$

c $\int_{-1}^{1}(2x^4+x^3-x^2-1)dx$

d $\int_{0}^{3}(2x^2+3x-4)dx$ e $\int_{1}^{5}(2x+7)dx$

8 a i 270 ii 61.5 iii 208.5

b $\int_{2}^{6}x^3dx - \int_{2}^{6}x^2dx = 270-61.5 = 208.5 =$

$\int_{2}^{6}(x^3-x^2)dx$

9 a 20.26 b −20.26 c $\int_{3}^{1}x^3dx = -\int_{1}^{3}x^3dx$

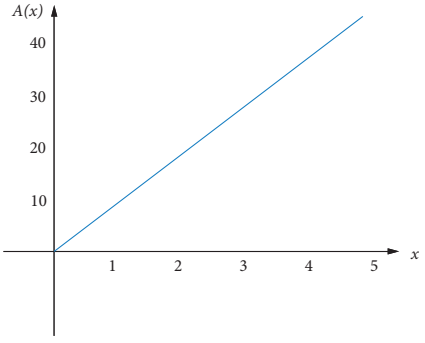
d $\int_{a}^{b}f(x)dx + \int_{b}^{a}f(x)dx = \int_{a}^{a}f(x)dx = 0,$

so $\int_{a}^{b}f(x)dx = -\int_{b}^{a}f(x)dx$

10 30.75 m

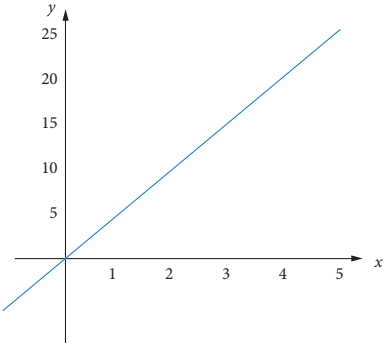
4.05

1 a

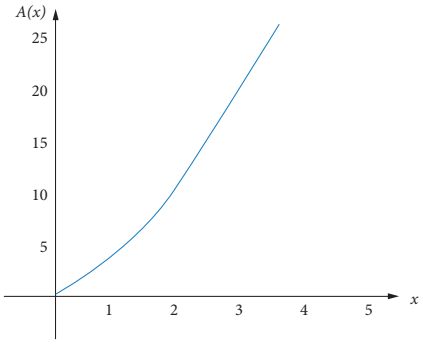


b $A(x) = 9x$

2 a

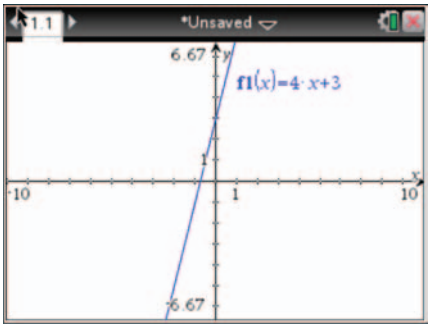


b

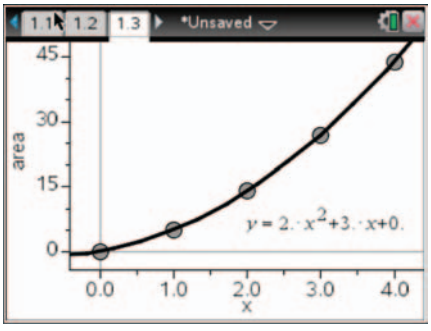


c $A(x) = 3x^2$

3 a



b



c $A(x) = 2x^2 + 3x$

4 a 72 b 20.25 c 10.67

d 8192 e 625

5 a 8.67 b 30 c 576.4

d 43.75 e 71.67

6 a 7765.33 b 104 857.5 c 10 d 10.5

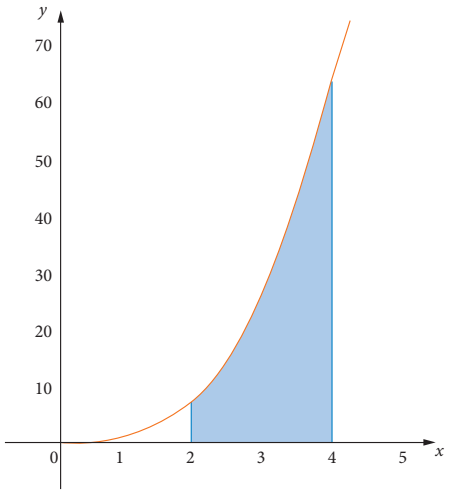
e 16.25 f 204.6 g 10.5 h 48 008

i 102.3 j 7654.5

7 a i 25 m/s ii 100 m/s

b 41.67 m c 291.67 m

8 a



b $\int_2^4 x^3 dx$ c 60 units^2

9 a 190.833 b 190.833 m

c 123.33 m

10 a 8 m/s^2 b 4 m/s c 60 m/s

4.06

1 a 16 b 128 c 15 d 4

e 16 f 52 g $\frac{1}{4}$ h $-1\frac{1}{2}$

i 25 j $\frac{13}{15}$

2 a $1\frac{1}{3}$ b 2 c 6

d 50 e $3\frac{5}{6}$

3 a $e^4 - 1$ b $5e(e^2 - 1)$ c $2e^2$

d $e^5 - e - 4$ e $60 - e^4 + e^2$

4 a $\frac{1}{\sqrt{2}}$ b $\frac{\sqrt{3}-1}{2}$ c 6

d $\frac{2(\sqrt{2}-1)}{\sqrt{2}} = 2 - \sqrt{2}$ e 7

5 a $\frac{\pi^2 + 4}{2}$ b $\sqrt{2} - 1$

c $\frac{3\sqrt{3} + \pi - 3}{6}$ d $\frac{3\sqrt{3} + 2}{2}$

e 27.53

6 a 0 b $-\frac{\sqrt{3}}{2}$ c $-1\frac{1}{2}$

d -1 e $-\sqrt{3} - 2$

7 a i $\int_0^5 (2x - 1)dx$ ii 20

b i $\int_0^4 (e^x + x)dx$ ii $e^4 + 7$

c i $\int_0^{\frac{\pi}{6}} [\cos(x) - 2\sin(x)]dx$ ii $\frac{2\sqrt{3} - 3}{2}$

8 a $\frac{1}{\cos^2(x)}$ b $\sqrt{3} - 1$

9 a $4e^{4x}$ b $e^{12} - 1$ c $\frac{e^{12} - 1}{4}$

10 a -5 cm/s b $x = t^3 + t^2 - 5t + 1$

c 126 cm d 20 cm/s^2

4.07

1 93

2 93

3 156

4 48

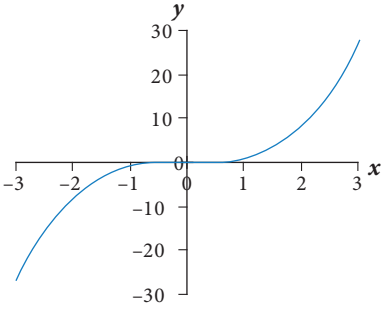
5 114

6 $28\frac{1}{2}$

7 6474

8 $109\frac{1}{3}$

9 a



b -4 c 4 d 0 e 8

f Because one half is below the axis and the other half is above, they cancel out in the integral.

10 a Negative

b $-30\frac{2}{3}$

c $30\frac{2}{3}$, since an area must be positive.

CHAPTER 4 REVIEW

1 B

2 D

3 D

4 D

5 C

6 a 600 km b Area = $75 \times 8 = 600$ km

7 12 m

8 3.08 units^2

9 a i 1.75 units^2 ii 3.75 units^2

b 2.5872 units^2

10 a 2.1875 units^2 b 3.1875 units^2

c 2.65625 units^2

11 60.84 units^2

12 403.75

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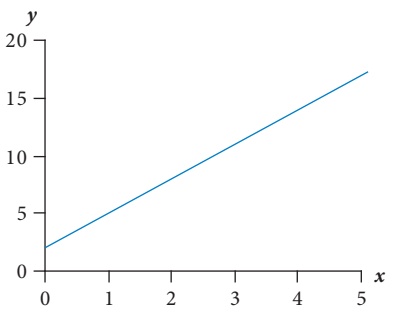
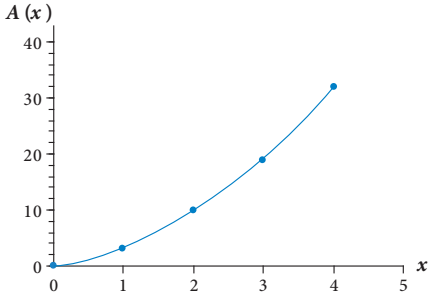
13 a



b $A(x)$



c $y = \frac{3x^2}{2} + 2x$

14 a i 7.25 ii 45.5 iii 52.75

b $\int_1^2 (2x^2+1)dx + \int_2^4 (2x^2+1)dx = 7.25 + 45.5$

$= 52.75 = \int_1^4 (2x^2+1)dx$

15 a i 166.204 ii 997.222

b $\int_2^8 6x^2 dx = 1020.2616 = 6 \times 170.0436 = 6\int_2^8 x^2 dx$

16 a i 2.92 ii 2.75 iii 5.67

b $\int_1^2 x^3 dx + \int_1^2 2x\,dx = 2.92 + 2.75 = 5.67$

$= \int_1^2 (x^3 + 2x)dx$

17 a 4 b 4 c 10.5 d 2.5

18 $3(e^7 - 1)$

19 a $\frac{2-\sqrt{2}}{2}$ b $\frac{\sqrt{3}-1}{2}$

20 a $32\frac{2}{3}$ units^2 b 93 units^2

21 a 1.62 cm/s b 2.52 cm c 0.656 cm

22 a $\frac{1-x}{e^x}$ b $\frac{1}{e}$

23 a Negative b Positive c 0

d −4 e 8

f Because the part from 0 to 2 is below the axis and the part from 2 to 4 is above the axis, but anti-symmetrical.

5.01

1 b, c and e

2 E

3 E

4 a Yes b No c No d Yes

e Yes f No g No

5 a $\frac{1}{2}$ b $\frac{1}{8}$ c $\frac{1}{4}$

6 a $\frac{6}{25}$ b $\frac{2}{9}$ c 0.24

d 0.21 e $\frac{3}{16}$

7 a About 0.4899 b $\frac{\sqrt{2}}{3}$

c $\frac{2\sqrt{6}}{25} \approx 0.1960$ d 0.4

e $\frac{\sqrt{14}}{9}$

8 $\frac{6\,436\,343}{9\,765\,625} \approx 0.659\,082$

9 $\frac{\sqrt{2}}{3}$

10 $\frac{12}{169}$

11 $\frac{1024}{3125} = 0.327\,68$

12 About 74% (0.7371…)

5.02

1 D

2 B

3 D

4 B

5 A

6 **a**, **c**, **d** and **e** are geometric distributions.

7 0.0439

8 0.7037

9 a 0.0655 b 0.738 c 4

10 a 0.75 b 0.5720

11 a 0.4502 b 9

12 a 0.0531 b 0.4695

13 a 0.25 b 0.25 c 0.875

14 0.00293

15 0.059

16 a 0.2373

b 2 (you can expect that your third fraudulent return will be detected)

17 About 0.0117

5.03

1 D

2 B

3 E

4 **b**, **c**, **e** and **g**

5 a 0.3241 b 0.2618 c 0.2048

d 0.0004 e 0.3894

6 a $p = 0.5, q = 0.5, n = 10, x = 2$

b $p = 0.15, q = 0.85, n = 20, x = 0$

c $p = \frac{3}{5}, q = \frac{2}{5}, n = 15, x = 12$

d $p = 0.11, q = 0.89, n = 9, x = 8$

e $p = 0.25, q = 0.75, n = 7, x = 4$

7 **a** 7 **b** 0.8

c

x	0	1	2	3
$P(X=x)$	0.0000	0.0004	0.0043	0.0287

x	4	5	6	7
$P(X=x)$	0.1147	0.2753	0.3670	0.2097

8 **a** 7 **b** 0.15

c

z	0	1	2	3
$P(Z=z)$	0.3206	0.3960	0.2097	0.0617

z	4	5	6	7
$P(Z=z)$	0.0109	0.0012	0.0001	0.0000

9 0.0781

10 The value of p in question **7** is much higher than the value of p in question **8** and the probabilities of more successes in question **7** are much higher than the corresponding probabilities in question **8**. Similarly, the probabilities of more failures in question **7** are much lower than the corresponding probabilities in question **8**.

5.04

1 D
2 B
3 D
4 D
5 B

6 **a** 0.2903 **b** 0.5801 **c** 0.0188

7 **a** 0.0864 **b** 0.0334 (0.033 378...)
c 0.0001 (0.000 107…) **d** 0.8803

8 **a** 0.0002 **b** 0.3810 **c** 0.0073 **d** 0.8064

9 **a** 0.0650 **b** 0.1216
c 0.9770 **d** 0.0001 (0.000 149…)

10 **a** 0.2090 **b** 0.1239 **c** 0.8936

11 **a** 0.0024 **b** 0.0993 **c** 0.1028

12 **a** 3 **b** 1, 5 **c** 2, 4

13 **a** 2 **b** 4 **c** 2

INVESTIGATION: THE EFFECT OF CHANGING *n* AND *p* ON A BINOMIAL DISTRIBUTION HISTOGRAM

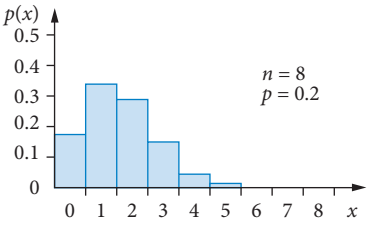
A As p increases from 0.2 to 0.5 to 0.8 the histograms change from being skewed right to symmetrical to skewed left.

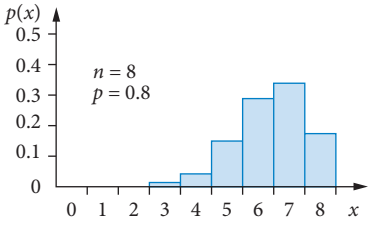
B As n increases, the graph becomes 'smoother' and the amount at the ends decreases.

5.05

1 C
2 D

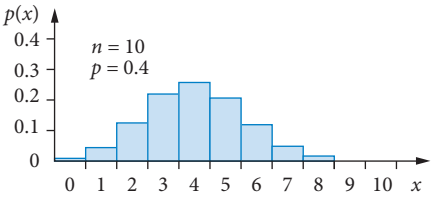
3 **a**

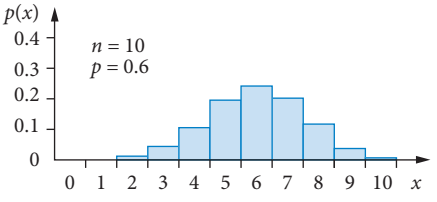




b The graphs are mirror images of each other, skewed positively and negatively respectively.

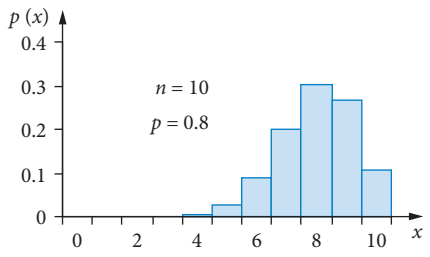
4 **a**





b The graphs are mirror images of each other, skewed positively and negatively respectively.

5 **a**



b 8

6 **a**

x	0	1	2	3	4	5
$P(X=x)$	0.0313	0.1563	0.3125	0.3125	0.1563	0.0313

$E(X) = 2.5$, $SD(X) \approx 1.118$

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b

x	0	1	2	3
$P(X=x)$	0.0576	0.1977	0.2965	0.2541

x	4	5	6	7	8
$P(X=x)$	0.1361	0.0467	0.01	0.0012	0.0001

$E(X) = 2.4$, $SD(X) \approx 1.296$

c

x	0	1	2	3
$P(X=x)$	0.0 …	0.0 …	0.0 …	0.0 …

x	4	5	6	7
$P(X=x)$	0.000	0.0033	0.0155	0.0532

x	8	9	10	11
$P(X=x)$	0.1329	0.2362	0.2835	0.2062

x	12
$P(X=x)$	0.0687

$E(X) = 9.6$, $SD(X) = 1.386$

d

x	0	1	2	3
$P(X=x)$	0.0003	0.0035	0.0212	0.0743

x	4	5	6	7
$P(X=x)$	0.1672	0.2508	0.2508	0.1612

x	8	9
$P(X=x)$	0.0605	0.0101

$E(X) = 5.4$, $SD(X) \approx 1.470$

7 **a** $E(X) = 0.7$, $SD(X) \approx 0.794$
b $E(X) = 6.3$, $SD(X) \approx 0.794$
c $E(X) = 13$, $SD(X) \approx 2.133$
d $E(X) = 10.2$, $SD(X) \approx 2.595$

8 0.1 or 0.9

9 $\frac{5}{8}$

10 **a** The experiment is binomial.
b About 5 (4.6). **c** About 95%.

11 $n = 25$ and $p = 0.2$, so $p(5) \approx 0.196$

12 About 100.

13 625

14 **a** 168 **b** About 10.04.

5.06

1 **a** $n = 3, p = \frac{1}{4}, q = \frac{3}{4}$ **b** 0.016
c 0.422 **d** 0.156

2 **a** 0.296 **b** 0.988 **c** 0.593

3 **a** $\frac{5}{16} = 0.3125$ **b** $\frac{11}{32} = 0.343\,75$

4 $\frac{11}{16} = 0.6875$

5 **a** $1.708 \times 10^{-6} \approx 0$ **b** 0.679

6 **a** 0.663 **b** 0.051 **c** 0.337

7 **a** 0.012 **b** 0.16 **c** 0.841

8 **a** 0.000 061 **b** 0.133 **c** 0.173 **d** 0.244

9 0.895

10 **a** $n = 3, p = 0.2, q = 0.8$ **b** 0.008
c 0.512 **d** 0.488

11 **a** 0.555 **b** 4 times

12 **a** 0.2533 **b** 0.8596

13 There is a 95% chance that from 984 to 1056 students from the new community will attend government schools.

14 0.262

15 0.633

16 0.0005

17 **a** **i** 0.1937 **ii** 0.6513 **b** 12

18 **a** 0.147 **b** 0.832

19 $\frac{577}{117649}$

20 **a** **i** 0.1176 **ii** 0.8824 **b** 5

21 **a** 0.276 **b** 0.003 **c** 0.180

22 0.678

CHAPTER 5 REVIEW

1 B
2 C
3 D
4 E
5 A
6 E
7 B
8 D
9 D
10 **c**, **e** and **g**

11 $\frac{8}{45}$

12 $\frac{5}{36}$

13 0.0658

14 **a** 0.1209 **b** 0.8824 **c** 2.333...

15 **a** 0.0223 **b** 0.1024

16 **a**, **c**, **e**

17 **a** 0.3456 **b** 0.0604 **c** 0.0171

18 **a** 8 **b** 0.45

c

x	0	1	2	3
$P(X=x)$	0.0084	0.0548	0.1569	0.2568

x	4	5	6	7	8
$P(X=x)$	0.2627	0.1719	0.0703	0.0164	0.0017

19 **a** 0.003 346 **b** 0.8309 0.1061

20 0.2424

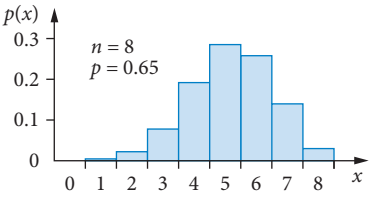
21 **a** 0.1354 **b** 0.9994 **c** 0.0279

22 **a** 0.0743 **b** 0.2508 **c** 0.9962

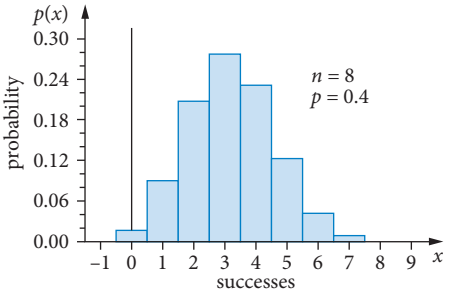
23 0.784

24 **a** 0.206 **b** 0.144 **c** 0.000 03
d 0.985 **e** 0.9166

25 0.5398

26 

The graph is negatively skewed.

27 a 

b Most likely number of successes = 3.

28 a $\mu = 1.6, \sigma = 1.13$ **b** $\mu = 7, \sigma = 1.45$
c $\mu = 8.25, \sigma = 1.9268$

29 a 0.016 **b** 0.022 **c** 0.170

30 a 0.0029 **b** 0.1937 **c** 0.8900

31 a 0.107 **b** 0.268 **c** 0.879

32 a 0.087 **b** 0.681 **c** 0.823

33 a 0.463 **b** 0.135 **c** 0.171 **d** 0.964

34 0.3327

35 a About 5. **b** 0.9678
c There is about a 97% chance that from 1 to 8 women in the study group will suffer from iron deficiency.

36 0.0858

37 a i 0.1374 **ii** 0.5608 **b** 12

INVESTIGATION: BASIC INTEGRALS

$(n+1)x^n$

x^n

$\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\cos(x)$

$\int \cos(x) dx = \sin(x) + c$

$\sin(x)$

$\int \sin(x) dx = -\cos(x) + c$

e^x

$\int e^x dx = e^x + c$

$f'(ax+b)$

$\int \frac{f(ax+b)}{a} dx = F(ax+b) + c$ where $F(x)$ is a primitive of $f(x)$

6.01

1 B
2 D
3 A

4 a $\frac{x^2}{2} + c$ **b** $\frac{x^3}{3} + c$ **c** $\frac{x^7}{7} + c$ **d** $\frac{2x^5}{5} + c$
e $-\frac{5}{2x^2} + c$ **f** $-\frac{3x^4}{4} + c$ **g** $\frac{5}{3x^3} + c$ **h** $\frac{2x^{\frac{3}{2}}}{3} + c$
i $\frac{10x^{\frac{3}{2}}}{3} + c$ **j** $\frac{x^6}{42} + c$ **k** $\frac{x^4}{20} + c$
l $-\frac{1}{8x^2} + c$ **m** $\frac{3x^{\frac{4}{3}}}{4} + c$ **n** $\frac{15x^{\frac{7}{5}}}{7} + c$
o $4x^{\frac{1}{4}} + c$ **p** $-\frac{2}{x^2} + c$ **q** $\frac{1}{x^5} + c$
r $20\sqrt{x} + c$ **s** $-9x^{\frac{2}{3}} + c$ **t** $-\frac{16}{\sqrt{x}} + c$

5 a $\frac{1}{2}e^{2x} + c$ **b** $\frac{1}{4}e^{4x} + c$ **c** $-e^{-x} + c$
d $\frac{1}{5}e^{5x} + c$ **e** $-\frac{1}{2}e^{-2x} + c$ **f** $\frac{1}{4}e^{4x+1} + c$
g $-\frac{3}{5}e^{5x} + c$ **h** $\frac{1}{2}e^{2t} + c$ **i** $\frac{5}{4}e^{4x} + c$
j $3e^{-2x} + c$ **k** $8e^{\frac{x}{2}} + c$ **l** $-18e^{-\frac{x}{3}} + c$

6 a $\sin(x) + c$ **b** $-\cos(x) + c$
c $-\frac{1}{3}\cos(3x) + c$ **d** $\frac{1}{7}\cos(7x) + c$
e $\sin(x+1) + c$ **f** $-\frac{1}{2}\cos(2x-3) + c$
g $\frac{1}{2}\sin(2x-1) + c$ **h** $-8\cos\left(\frac{x}{2}\right) + c$
i $-\cos(3-x) + c$ **j** $12\sin\left(\frac{x}{4}\right) + c$
k $\cos(\pi - x) + c$ **l** $\sin(x+\pi) + c$
m $5\cos\left(\frac{2x}{5}\right) + c$ **n** $\frac{16}{7}\sin\left(\frac{7x}{4}\right) + c$
o $\frac{6}{\pi}\sin\left(\frac{\pi x}{3}\right) + c$ **p** $-\frac{2\pi}{3}\cos\left(\frac{-3x}{\pi}\right) + c$

7 a $\frac{(x+1)^5}{5} + c$ **b** $\frac{(5x-1)^{10}}{50} + c$
c $\frac{(3y-2)^8}{24} + c$ **d** $\frac{(4+3x)^5}{15} + c$
e $\frac{(7x+8)^{13}}{91} + c$ **f** $-\frac{(1-x)^7}{7} + c$
g $\frac{\sqrt{(2x-5)^3}}{3} + c$ **h** $-\frac{2(3x+1)^{-3}}{9} + c$
i $-3(x+7)^{-1} + c$ **j** $-\frac{1}{16(4x-5)^2} + c$
k $\frac{3\sqrt[3]{(4x+3)^4}}{16} + c$ **l** $-2(2-x)^{\frac{1}{2}} + c$
m $\frac{2\sqrt{(t+3)^5}}{5} + c$ **n** $\frac{2\sqrt{(5x+2)^7}}{35} + c$
o $-\frac{1}{15(5x-4)^3} + c$ **p** $-\frac{3}{8(4x-3)^4} + c$

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8 $y = -\frac{3}{2}x^2 + 8$

9 $f(x) = \frac{3}{2}e^{2x} + 4$

10 $\frac{1}{2}e^{x^4} + c$

11 $\frac{1}{4}(4x^2 + 1)^3 + c$

6.02

1 C

2 C

3 A

4 D

5 a $\frac{m^2}{2} + m + c$ b $\frac{t^3}{3} - 7t + c$

c $\frac{h^3}{3} + 5h + c$ d $\frac{y^2}{2} - 3y + c$

e $x^2 + 4x + c$ f $\frac{b^3}{3} + \frac{b^2}{2} + c$

g $\frac{a^4}{4} - \frac{a^2}{2} - a + c$ h $\frac{x^3}{3} + x^2 + 5x + c$

i $x^4 - x^3 + 4x^2 - x + c$ j $x^6 + \frac{x^5}{5} + \frac{x^4}{2} + c$

k $\frac{x^8}{8} - \frac{3x^7}{7} - 9x + c$ l $\frac{x^4}{2} + \frac{x^3}{3} - \frac{x^2}{2} - 2x + c$

m $\frac{x^6}{6} + \frac{x^4}{4} + 4x + c$ n $\frac{4x^3}{3} - \frac{5x^2}{2} - 8x + c$

o $\frac{3x^5}{5} - \frac{x^4}{2} + \frac{x^2}{2} + c$ p $\frac{3x^4}{2} + \frac{5x^3}{3} - 4x + c$

q $-x^{-3} - \frac{x^{-2}}{2} - 2x^{-1} + c$ r $\frac{14x^{\frac{5}{2}}}{5} - 2x^2 + 9x^{\frac{2}{3}} + c$

6 a $\frac{x^4}{4} - x^3 + x^2 + c$ b $x - 2x^2 + \frac{4x^3}{3} + c$

c $\frac{x^3}{3} + \frac{3x^2}{2} - 10x + c$

d $-\frac{4}{x} - x + \frac{3}{2x^2} - \frac{7}{4x^4} + c$ e $\frac{y^3}{3} + \frac{y^{-6}}{6} + 5y + c$

f $\frac{t^4}{4} - \frac{t^3}{3} - 2t^2 + 4t + c$ g $\frac{2\sqrt{x^3}}{3} + x + c$

h $\frac{-1}{x} - \frac{3}{2x^2} + \frac{10}{3x^3} + c$ i $\frac{4x^{\frac{5}{2}}}{5} - \frac{8x^{\frac{3}{2}}}{3} + 6\sqrt{x} + c$

7 a $y = x^2 - 5x + 2$ b $y = 2x^{\frac{3}{2}} - 2x^2 + 10$

c $y = x^3 - \frac{x^2}{2} + 2x - 10$

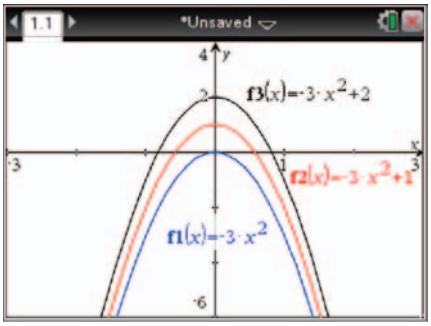
8 a $y = 3x^2 - x + 5$ b $y = -2x^2 + 7x + 10$

c $y = \frac{-3}{x} + 2x + 6$ d $y = 4\sqrt{x} + \frac{3x^2}{2} - \frac{5}{2}$

e $y = \frac{3x^{\frac{4}{3}}}{4} + 2x^3 - 10x + \frac{1}{4}$

9 a $-3x^2$

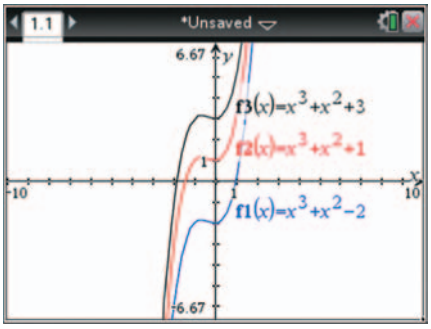
b $f1(x) = -3x^2$, $f2(x) = -3x^2 + 1$, $f3(x) = -3x^2 + 2$

c 

The functions are vertical translations of each other.

10 a $x^3 + x^2$

b $f1(x) = x^3 + x^2 - 2$, $f2(x) = x^3 + x^2 + 1$, $f3(x) = x^3 + x^2 + 3$

c 

The functions are vertical translations of each other.

11 $f(2) = 1$

12 $f(x) = \frac{2}{3x^{\frac{3}{2}}} - \frac{16}{x} + 66\frac{2}{3}$

6.03

All areas are in square units.

1 12.5

2 18

3 a $\int_{-1}^{2.5}(-2x + 5)dx$ b $\int_{1}^{5}(x + 5)dx$

c $\int_{-3}^{-1}(x^2)dx$ d $\int_{2}^{4}(2x^2)dx$

e $-\int_{-1}^{1}(-e^x)dx$ f $-\int_{3}^{5}(x^3 - 7x^2 + 4x + 11)dx$

g $\int_{0}^{\pi}[3\sin(x)]dx$ h $\int_{2}^{8}(-x^3 + 10x^2 - 5x)dx$

4 a $508\frac{1}{2}$ b 48 c 12 480 d $624\frac{6}{7}$

e 6144 f $95\frac{5}{6}$ g 480 h -1

i $44\frac{1}{3}$ j 50 k $52\frac{2}{3}$ l $1\frac{1}{4}$

5 a $\frac{7}{800}$ b $\frac{1}{3}$ c $\frac{-6}{25}$

d $\frac{13}{27}$ e $\frac{7}{32}$ f $2(\sqrt{3} - \sqrt{2}) \approx 0.6357$

6 a $\frac{2}{3}$ b $\frac{3}{8}$ c $\frac{2}{5}$

d About 0.0672 e $\frac{2}{9}$ f $\frac{26}{9}$

7 a $\frac{1}{3}\left(e^{\frac{3\pi}{4}} - e^{-\frac{3\pi}{4}}\right)$ b $5e^3(e^5 - 1)$

c $\frac{1}{5}(e^5 - 1)$ d $e^{-2} - 1 = \frac{1}{e^2} - 1$

e $\frac{2}{3}e^7(e^9 - 1)$ f $19 - \frac{1}{2}e^4(e^2 - 1)$

g $\frac{1}{2}e^4 + 1\frac{1}{2}$ h $e^2 - e - 1\frac{1}{2}$

i $\frac{1}{2}e^6 + e^{-3} - 1\frac{1}{2}$

8 a 54 244.90 b 0.32 c 268.29

d 37 855.68 e 346.85 f 755.19

9 a 2 b $\frac{1}{\sqrt{2}}$ c $\frac{2}{\sqrt{2}} = \sqrt{2}$ d $-\frac{1}{3}$

e $\frac{1}{\pi}$ f $\frac{1}{2}$ g $\frac{3}{4}$ h $-\frac{1}{5}$

10 a $127\frac{1}{3}$ b $49\frac{1}{2}$ c 510

d $3(1 - 9^{-6}) \approx 3.00$ e $\frac{4}{5}\left(8\sqrt[4]{8} - 1\right) \approx 9.96$

f $\frac{19}{324} \approx 0.06$ g $2(e - e^{-3}) \approx 5.34$

h $\frac{10}{33} \approx 0.30$ i 0

j $e^6 - e^4 - 260 \approx 88.83$

k $\frac{1}{6}\left(125 - 17\sqrt{17}\right) \approx 9.15$ l −14 560

m $2\sqrt{2} \approx 2.83$ n 0 o 0

11 a The function and indefinite integral are not defined at the limit $x = 0$.

b The function and indefinite integral are not defined at the limit $x = 5$.

c The function and indefinite integral are not defined at the limit $x = -1$.

12 The function and indefinite integral are not defined at the value $x = 2$, which is included in the limits of integration.

13 The function and indefinite integral are not defined at 0, which is included in the limits of integration.

14 $\frac{d}{dx}xe^{x^2} = e^{x^2} + 2x^2e^{x^2}$, so $\int_0^1 (2x^2e^{x^2} + e^{x^2})dx = e$.

15 1275 m^3

INVESTIGATION: SIMPLE POWERS

1 2, 0, $\frac{2}{3}$, 0, $\frac{2}{5}$, 0

2 For n even, $\int_{-1}^{1} x^n dx = \frac{2}{n+1}$ and for n odd $\int_{-1}^{1} x^n dx = 0$.

3 The even powers have the same area either side of 0, but the odd powers have opposite areas.

4 The results are similar as the odd powers are all 0.

6.04

All areas are in square units.

1 D

2 B

3 E

4 D

5 E

6 a $-\int_0^3 f(x)dx + \int_3^6 f(x)dx$

b $\int_{-9}^{-6} g(x)dx - \int_{-6}^{-2} g(x)dx$

c $-\int_{-5}^{4} h(x)dx$

d $\int_{-4}^{-1} k(x)dx - \int_{-1}^{3} k(x)dx$

e $-\int_{-5}^{-2} m(x)dx + \int_{-2}^{2} m(x)dx$

f $\int_1^3 p(x)dx - \int_3^5 p(x)dx + \int_5^6 p(x)dx$

7 a $7\frac{1}{2}$ b 39 c 74

d $67\frac{1}{2}$ e $43\frac{1}{2}$

8 $2(e^2 + e^{-2} - 2) \approx 11.05$

9 6

10 a $38\frac{1}{4}$ b $73\frac{5}{6}$

11 a 32 b $24\frac{35}{54}$ c $\frac{2}{3}$

12 a $49\frac{1}{3}$ b $31\frac{1}{4}$

13 4

14 $e^3 - e^{-3} \approx 20.04$

15 $\frac{1}{3}$

16 $\pi \approx 3.14$

17 $\frac{1}{4}(e - e^{-3})$

18 $3 - \frac{1}{e^2} \approx 2.86$

19 $\frac{1}{3\sqrt{2}} = \frac{\sqrt{2}}{6}$

20 a 14.91 b 21.08 c 94.03

21 a 20 b $132\frac{3}{4}$

22 a $4\frac{1}{2}$ b $14\frac{7}{24}$

23 a $49\frac{1}{3}$ b $123\frac{11}{32}$ c $18\frac{2}{3}$

24 $93 392

25 750 J

6.05

All areas are in square units.

1 C
2 B
3 D
4 A
5 $20\frac{5}{6}$
6 a $10\frac{2}{3}$ b $24\frac{1}{2}$ c $7\frac{1}{3}$
d 36 e 3.267... f 18.42
7 $1\frac{1}{3}$
8 $10\frac{2}{3}$
9 $166\frac{2}{3}$
10 a 9 b 36 c $83\frac{1}{3}$
d $2\frac{1}{4}$ e 18
11 $\frac{1}{12}$
12 $2\frac{2}{3}$
13 $\frac{2}{3}$
14 $\frac{5}{12}$
15 $\frac{2}{3}$
16 $\pi - 2$
17 a $58\frac{2}{3}$ b 72 c $21\frac{1}{12}$
d $21\frac{1}{12}$ e $78\frac{1}{12}$
18 $\frac{1}{3}$
19 $1.996 \approx 2$ kg
20 0.8 m^2
21 About 22 775 000 L ≈ 23 ML.
22 a 268 m^2 b 6700 m^3
23 87.6 m^2, \$4599

6.06

1 B
2 D
3 The change in the height (in cm) of the pile from time $t = 0$ to time $t = 10$ (the first 10 hours).
4 The increase in the number of bacteria in the culture from time $t = 2$ to time $t = 8$ (from the start of the second hour to the end of the eighth hour).
5 $150 + \int_0^4 P'(t)dt$
6 a About 7566 L.
b About 1024 L.
c The rate of leakage decreases as time goes on.
7 a 21 000 b 3 months
8 \$625 000
9 $R(x) = 12x - x^3 + 2x^2$; \$16 000
10 407.36 L
11 $R(x) = x(10 - 0.001x)$; $C(x) = 7000 + 2x$; Total profit = \$0, break-even point
12 $C(x) = 8x^2 - x^3 + 5x + 400$
13 $C(x) = \frac{1}{4}x^2 + 5000$

6.07

1 The displacement of the particle from time = 2 s to time = 10 s.
2 50.4 m
3 Displacement = 155 cm, position = 153 cm to the right of the origin.
4 3 cm to the left of the origin.
5 a $\left(\sqrt{3}+3\right)$ m to the right of the origin
b $-4\sqrt{3}$ m/s^2
6 a 277.5 m b −29 m/s c 2.04 s
7 a 10 m b $2\frac{6}{7}$ s c −14 m/s
8 30.63 m
9 225 km
10 About −743 cm
11 893 m
12 a $5e^{45}$ m/s b e^{30} m
13 490 m
14 a 6.3 m/s b 12.25 m
15 a 7 cm/s^2 b $15\frac{1}{3}$ cm to the right of the origin.
16 a −1 cm/s b $\frac{2-\pi}{8}$ cm c $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \ldots$ s

CHAPTER 6 REVIEW

All areas are in square units.

1 E
2 E
3 A
4 C
5 A
6 D
7 D
8 C
9 B
10 C
11 A
12 a $\frac{y^4}{4} - y^3 + 2y^2 + y + c$ b $\frac{1}{n} + c$ c $\frac{2}{x} + c$
d $\frac{1}{2}(3x^3 - 2x + 4)^2 + c$ e $-\cos(x) + c$
f $-\frac{1}{2}\sin(6x) + c$ g $\frac{1}{2}\cos(10x) + c$
h $\frac{1}{3}e^{3t} + c$ i $-\frac{3}{2e^{2x}} + c$

j $-\frac{2}{(x-5)^2}+c$ k $-\frac{1}{18(2x+7)^3}+c$

l $\frac{\sqrt{(4x+7)^3}}{6}+c$

13 a $\frac{x^5}{5}+7x+c$ b $x^5-\frac{x^4}{2}+2x^2+c$

c $\frac{3}{2}x^4-\frac{8}{3}x^3-3x+c$

14 $y=3x^2-4x+2$

15 $f(x)=3\sqrt{x}+2$

16 a $\frac{1}{5}x^5-x^3+7x+c$ b $-\frac{(2-3x)^3}{9}+c$

c $\frac{6}{5}x^{\frac{5}{2}}-\frac{10}{3}x^{\frac{3}{2}}+4\sqrt{x}+c$

17 a 30 b $17\frac{1}{3}$ c $\frac{5}{6}$

d $\frac{2}{3}$ e $\frac{\sqrt{3}+1}{4}\approx 0.68$ f 0

18 $\int_1^4 f(x)dx-\int_{-2}^1 f(x)dx$

19 a 111 b $e^2+e-e^{-2}-e^{-1}\approx 9.60$

c 4.83

20 About 60.64.

21 26

22 4444

23 $32\frac{3}{4}$

24 $20\frac{5}{6}$

25 $57\frac{1}{6}$

26 28.98

27 The total change (°C) in temperature of the liquid in the first 5 minutes.

28 a 338.39 L b 185.71 L

29 a 1950 b 6 months

30 $R(x)=1500x-2x^2-x^3$; \$16 200

31 Displacement = 25 m; position = 27 m to the left of the origin.

32 a 41.6 m b −19 m/s c 3.06 s

33 $v=\frac{100-10(1+2t)^3}{3}$ cm/s

34 a $(4,-e)$ b $y=(4e^{-1})x-e-16e^{-1}$

c $(0,-e-16e^{-1})$ d $(0,-1)$

e $|-32e^{-1}-4|\approx 15.77$

35 a 6 m b 72 m^2

c 100.8 m^3 d 8 709 120 m^3

e 247.5 : 100.8 ≈ 2.46 : 1

36 32.26 m^2

37 a 31.5 m/s

b For $v=0$, $3t^2+2t+30=0$ has no real solutions, so $v\neq 0$.

38 $\sqrt{3}$

39 29.07 m^2, \$944.83

40 3.125 cm

41 4433 cm^2, 13 300 L

MIXED REVISION 2

Multiple choice

1 B

2 C

3 D

4 D

5 D

6 E

7 B

8 A

9 A

Short answer

1 a x^3-4x^2+5x+c b $-\frac{e^{-2x}}{2}+c$

c $\frac{\sin(2x)}{2}+c$ d $\frac{(2x-3)^{10}}{20}+c$

e $-5\cos\left(\frac{x}{5}\right)+c$

2 0.0988

3 $y=3x^2-5x-7$

4 a 6 b 0.4

c

x	0	1	2	3
$p(x)$	0.0467	0.1866	0.3110	0.2765

x	4	5	6
$p(x)$	0.1382	0.0369	0.0041

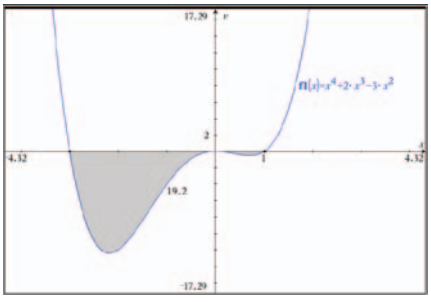
5 a $\frac{e^{18}-1}{3}$ b 14 c $\frac{3\left(\sqrt{2}+1\right)}{\sqrt{2}}$

6 21.76

Application

1 a $x=-3, 0, 1$

b



c 19.2 units^2

2 128 m^2

3 a About 0.0388 b About 0.3585

4 a $10\,482\times 3.71\times 10^7\approx 3.89\times 10^{11}$

b Number of atoms decayed.

c 693 s ≈ $11\frac{1}{2}$ min

5 a $3e^{30}$ cm/s b $x=e^{3t}$ c $a=9e^{3t}=9x$

6 11

9780170250290

7.01

1 **a** $5^2 = 25$ **b** $4^2 = 16$ **c** $5^3 = 125$ **d** $2^4 = 16$
e $3^1 = 3$ **f** $7^2 = 49$ **g** $2^7 = 128$ **h** $5^0 = 1$

2 **a** $8^{\frac{1}{3}} = 2$ **b** $4^{-\frac{1}{2}} = \frac{1}{2}$ **c** $7^{\frac{1}{4}} = \sqrt[4]{7}$
d $3^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{3}}$ **e** $2^{-\frac{3}{2}} = \frac{\sqrt{2}}{4}$ **f** $a^c = b$
g $c^{3m} = \sqrt{a}$

3 **a** $\log_7(49) = 2$ **b** $\log_3(27) = 3$
c $\log_2(16) = 4$ **d** $\log_5(125) = 3$
e $\log_{11}(1) = 0$ **f** $\log_2(1) = 0$

4 **a** $\log_5\left(\frac{1}{25}\right) = -2$ **b** $\log_4\left(\frac{1}{16}\right) = -2$
c $\log_{10}\left(\frac{1}{1000}\right) = -3$
d $\log_{\frac{1}{3}}\left(\frac{1}{81}\right) = 4, \log_3\left(\frac{1}{81}\right) = -4$
e $\log_{\frac{1}{4}}\left(\frac{1}{64}\right) = 3, \log_4\left(\frac{1}{64}\right) = -3$
f $\log_{\frac{1}{2}}\left(\frac{1}{8}\right) = 3, \log_2\left(\frac{1}{8}\right) = -3$
g $\log_6\left(\sqrt[3]{36}\right) = \frac{2}{3}$ **h** $\log_7\left(\sqrt[5]{343}\right) = \frac{3}{5}$
i $\log_a(m) = k$ **j** $\log_b(d) = 3$

5 **a** 6 **b** 2 **c** 4 **d** 3
e 3 **f** 0 **g** 1 **h** 5
i 5 **j** 5

6 **a** 4 **b** −3 **c** −2 **d** 4
e −7 **f** −9 **g** 4 **h** −1
i −3 **j** 3

7 **a** $\frac{1}{2}$ **b** $\frac{3}{2}$ **c** $\frac{3}{2}$
d $\frac{3}{2}$ **e** $\frac{2}{3}$

8 **a** $\frac{3}{2}$ **b** $\frac{3}{2}$ **c** $\frac{4}{3}$
d $\frac{5}{6}$ **e** $\frac{5}{3}$

9–11 Proofs

7.02

1 **a** 0 **b** 0 **c** 0 **d** 0
e 0 **f** 1 **g** 1 **h** 2
i 1 **j** 5

2 **a** undefined **b** undefined
c undefined **d** undefined
e undefined **f** undefined
g undefined

3 **a** 6 **b** 3 **c** $\frac{1}{2}$ **d** $\frac{3}{2}$
e $-\frac{1}{2}$ **f** 2 **g** 6

4 **a** 1 **b** 1 **c** 0
d 9 **e** 0 **f** $\log_5(50)$
g 1 **h** $\log_6\left(\frac{625}{64}\right)$

5 **a** 4 **b** $\frac{4}{3}$ **c** −4 **d** $-\frac{1}{2}$

6 **a** $\log_4(x^4)$ **b** $\log_7(x^2)$
c $\log_6\left(\frac{1}{x}\right)$ **d** $\log_2(x+2)^3$
e $\log_4(x-1)$ **f** 0

7 **a** $\log_{10}(12) + \log_{10}(a) - 1$
b $5\log_6(a) - 5\log_6(b)$
c $\frac{1}{5}\log_3(10) + \frac{3}{5}\log_3(x)$
d $\frac{2}{3}\log_4(x) + \frac{1}{3}\log_4(a) - 2\log_4(y)$

8 **a** 0.153 **b** 0.387 **c** 2.613

9 $\frac{1}{7}$

10 **a** 13 **b** 10 **c** −3

11–13 Proofs

7.03

1 **a** $\frac{\log(9)}{\log(4)}$ **b** $\frac{\log(6)}{\log(8)}$
c $\frac{\log(20)}{\log(2)} = 1 + \frac{1}{\log(2)}$ **d** $\frac{\log(200)}{\log(7)}$
e $\frac{\log(0.2)}{\log(9)}$

2 **a** 1.631 **b** 0.2789 **c** 1.683
d 0.4307 **e** 0.1262

3 **a** 6.644 **b** 1.585 **c** 2.640
d 16.01 **e** 15.75

4 **a** $x = 1.465$ **b** $x = 1.367$ **c** $x = 2.465$
d 2.861 **e** 0.774

5 **a** 3.051 **b** 0.2232 **c** 0.5767
d 0.6192 **e** 1.140

6 $4c + \frac{1}{b}$

7–8 Proofs

7.04

1 **a** $x = 1$
b $x = \log_5(0.8) \approx -0.1386$
c $x = \log_6(3) \approx 0.6131$
d $x = 2$
e $x = \log_8(5) \approx 0.7740$ or $x = \log_8(6) \approx 0.8617$

2 **a** $x = \frac{3\log(5)}{\log(9) - \log(5)}$
b $x = \frac{3\log(49)}{\log(49) - \log(8)}$
c $x = \frac{5[\log(350) + \log(4)]}{\log(350) - \log(4)}$

d $x = \dfrac{\log(15)}{\log(15) - 3\log(2)}$

e $x = \dfrac{2\log(17) + \log(7)}{2\log(7) - \log(17)}$

3 **a** $x = \dfrac{2\log(7)}{\log(7) - \log(4)} \approx 6.954$

b $x = \dfrac{4\log(4)}{\log(58) - \log(4)} \approx 2.074$

c $x = \dfrac{2\left[\log(5) + \log(46)\right]}{\log(46) - \log(5)} \approx 4.901$

d $x = \dfrac{3\log(5)}{2\log(6) - \log(5)} \approx 2.446$

e $x = \dfrac{\log(28) + 4\log(9)}{2\log(9) - \log(28)} \approx 11.41$

4 **a** $x = 83$ **b** $x = 55$ **c** $x = 6$

d $x = \dfrac{5}{8}$ or $x = 1\dfrac{1}{2}$ **e** $x = 5$ **f** $x = 3$

5 **a** $x = \dfrac{1}{10}, 1000$ **b** $x = \dfrac{1}{4}, 16$

c $x = \dfrac{1}{4}, 2$ **d** $x = \dfrac{1}{5}, 25$

e $x = 3, 27$ **f** $x = 390\,625, \dfrac{1}{125}$

6 **a** $x \approx 0.005\,53$ or $x \approx 3.90$

b $x \approx 1.62$ or $x \approx 19.7$

c $x \approx 0.225$ or $x \approx 3.37$

d No real solutions.

e $x \approx 0.001\,34$ or $x \approx 2.39$

7 0.0821 (8.21%)

8 45 years

INVESTIGATION: TRANSFORMING LOGARITHMS

a **i** $x > 0, y \in R$

ii $x = 1$

iii $x = 0$

iv $f(2) = 1$. It is the value of the function at the base of the function

b **i** $x > -3, y \in R$

ii $x = -2$

iii $x = -3$

iv $f(-1) = 1$. It corresponds to the base of the function

c **i** $x > -3, y \in R$

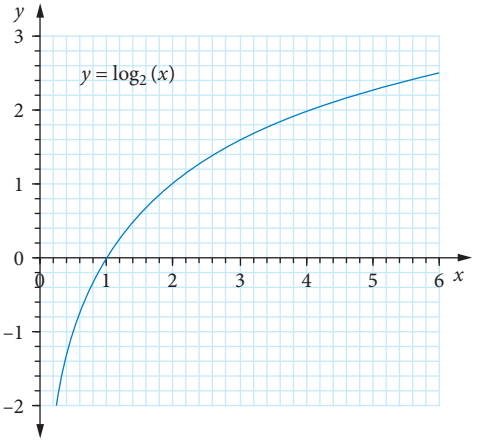
ii $x = 0$

iii $x = 2^{-4}$

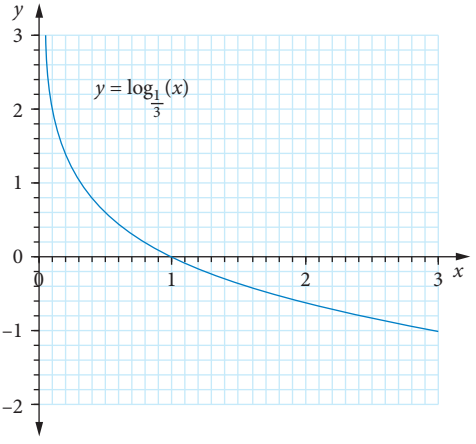
iv $f(2) = 5$. It corresponds to the base of the function.

7.05

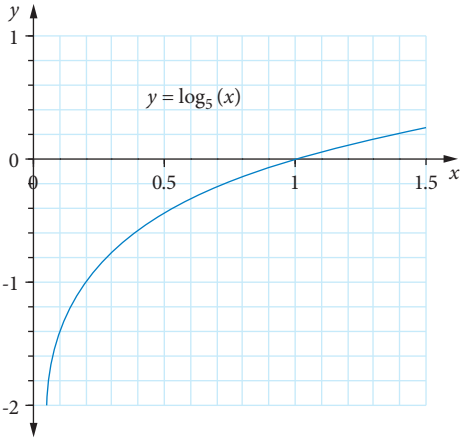
1 **a** $x \approx 4.9$



b $y \approx 0.2$



c $z \approx 0.27$



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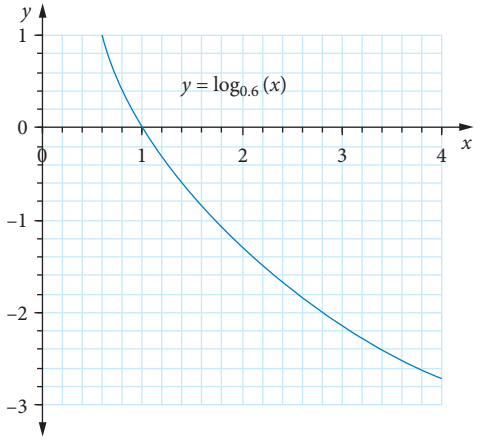
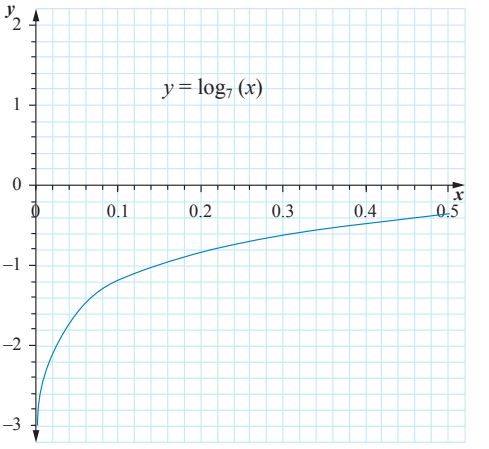
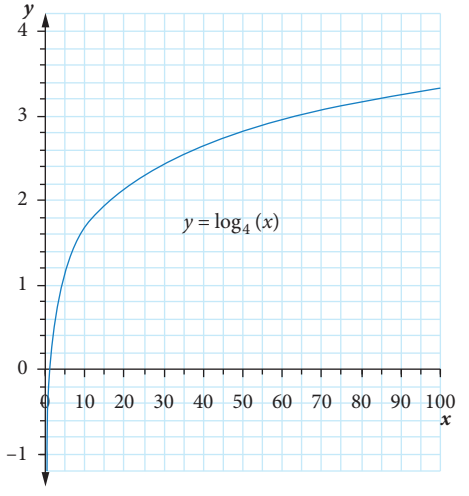
d $k \approx 2.4$



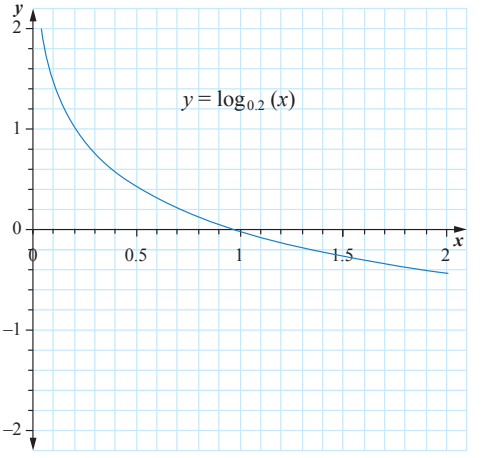
c $z \approx 0.04$



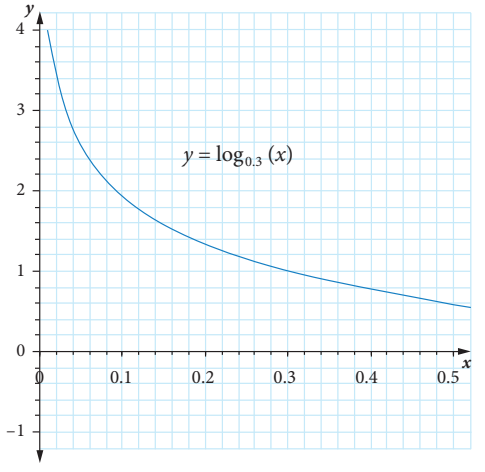
2 a $x \approx 75$



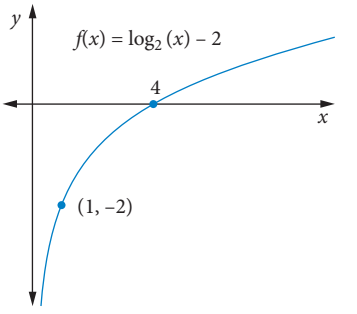
d $k \approx 1.9$



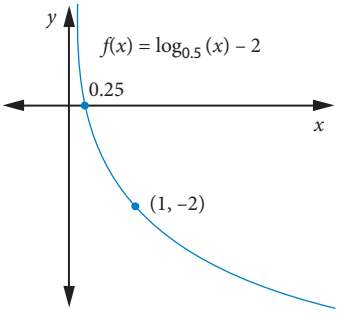
b $y \approx 0.05$



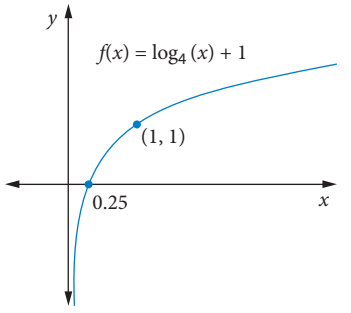
3 a



b



c



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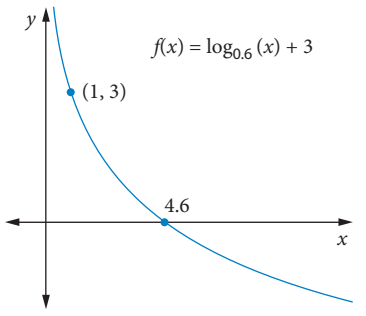

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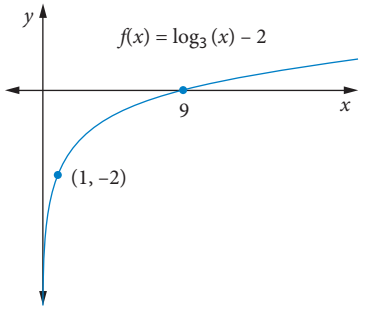
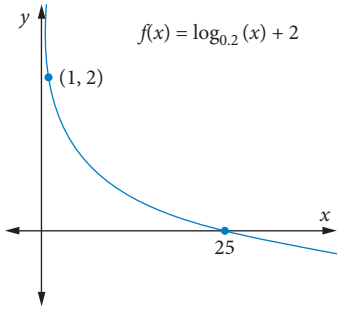
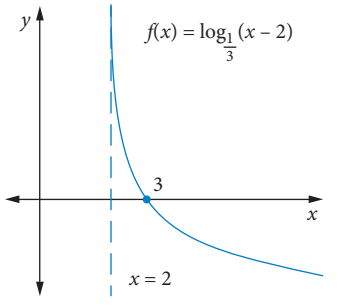


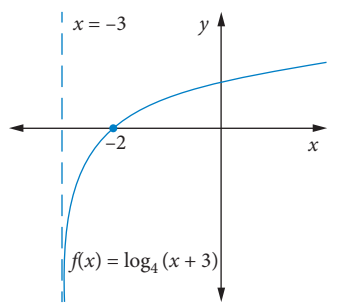
f



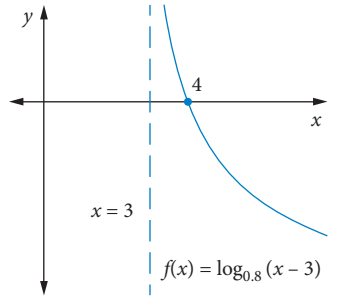
4 a



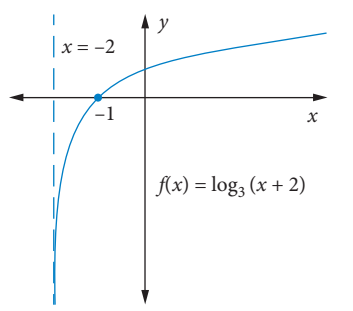
b



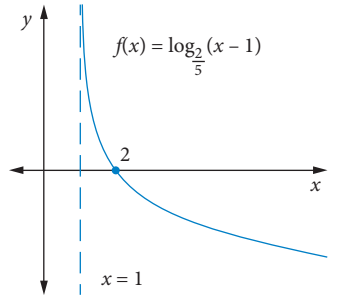
c



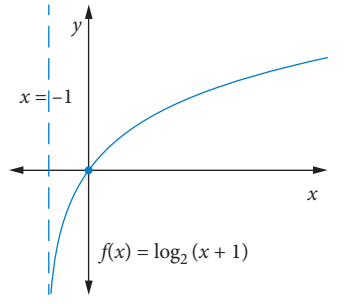
d



e



f



5 a $f(x) = \log_7(x) + 3$ b $f(x) = \log_{0.5}(x) - 2$
c $f(x) = \log_{\frac{1}{6}}(x) + 1$ d $f(x) = \log_3(x) - 4$

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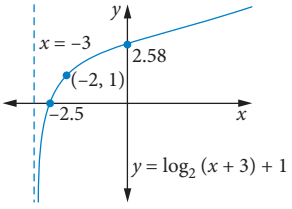
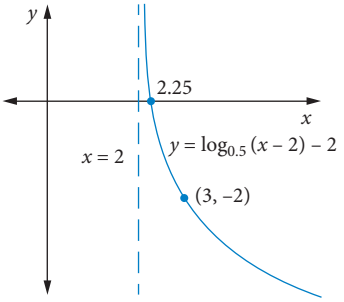
6 a $f(x) = \log_5(x+4)$ b $f(x) = \log_{0.3}(x-2)$
c $f(x) = \log_4(x+3)$ d $f(x) = \log_{0.8}(x-5)$

7 a $f(x) = \log_2(x+4)+3$ b $f(x) = \log_{0.1}(x-2)+1$
c $f(x) = \log_4(x+3)-4$ d $f(x) = \log_{0.6}(x-1)-2$

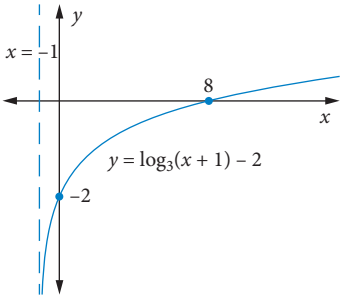
8 a



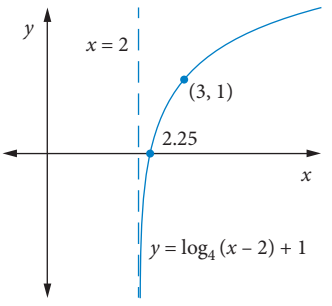
b



c



d



7.06

1 About 1700 km.

2 a 13 b 10 moles/litre

3 100 moles/litre

4 2488

5 3.16 watt/m^2

6 a 12.9686 b 1.570 65 cm

7 a 8 b 8.69×10^{10} kilowatt-hours

8 41.9886 litres

9 a $\sqrt[12]{2}$ b $440\sqrt[4]{2}$ Hz

7.07

1 a $\frac{dy}{dx} = \frac{1}{x}$ b $\frac{dy}{dx} = \frac{1}{x}$ c $\frac{dy}{dx} = \frac{3}{x}$

d $\frac{dy}{dx} = \frac{1}{x}$ e $\frac{dy}{dx} = \frac{6}{x}$ f $\frac{dy}{dx} = \frac{1}{x}$

g $\frac{dy}{dx} = \frac{4}{x}$ h $\frac{dy}{dx} = -\frac{2}{x}$

2 a $\frac{1}{x\ln(4)}$ b $\frac{1}{x\ln(10)}$ c $\frac{1}{x\ln(9)}$

d $\frac{1}{x\ln(2)}$ e $\frac{1}{x\ln(0.2)} = -\frac{1}{x\ln(5)}$

3 a $\frac{dy}{dx} = \frac{3}{3x-1}$ b $\frac{dy}{dx} = \frac{2}{2x+7}$

c $\frac{dy}{dx} = \frac{8}{4x-3}$ d $\frac{dy}{dx} = \frac{30}{6x+7}$

e $\frac{dy}{dx} = \frac{2}{2x+1}$ f $\frac{dy}{dx} = \frac{15}{5x-1}$

g $\frac{dy}{dx} = \frac{24}{4x-3}$ h $\frac{dy}{dx} = \frac{96}{8x-5}$

4 a $\frac{dy}{dx} = \frac{5}{x}$ b $\frac{dy}{dx} = \frac{3}{x}$

c $\frac{dy}{dx} = \frac{4x}{2x^2+1}$ d $\frac{dy}{dx} = \frac{32x}{8x^2-5}$

e $\frac{dy}{dx} = \frac{3x^2-4x+3}{x^3-2x^2+3x-4}$

f $\frac{dy}{dx} = \frac{24x^3-105x^4+3}{2x^4-7x^5+x}$

5 a $\frac{dy}{dx} = \frac{3}{2(3x+1)}$ b $\frac{dy}{dx} = \frac{7}{2(7x-5)}$

c $\frac{dy}{dx} = \frac{4}{3(4x+9)}$ d $\frac{dy}{dx} = \frac{1}{5(x-8)}$

e $\frac{dy}{dx} = \frac{12x}{3x-7}$

f $\frac{dy}{dx} = \frac{15}{5x-2}$

g $\frac{dy}{dx} = -\frac{1}{x+2}$

h $\frac{dy}{dx} = -\frac{3}{3x-5}$

i $\frac{dy}{dx} = \frac{12}{6x+1}$

j $\frac{dy}{dx} = \frac{5}{x-4}$

6 a $\frac{dy}{dx} = \frac{4x}{x^2+2}$

b $\frac{dy}{dx} = \frac{4x}{x^2-3}$

c $\frac{dy}{dx} = \frac{3(3x^2-2)}{x^3-2x+3}$

d $\frac{dy}{dx} = \frac{6(3x^2-3x+2)}{2x^3-3x^2+4x-1}$

7 **a** $2(x-2)\ln(x)+\dfrac{x^2-2x+1}{x}$

b $(3x^2+6x)\ln(x^3+3x^2+5)+3x(x+2)$

c $\ln(x)+1$

d $e^x\ln(x)+\dfrac{e^x}{x}$

e $\ln(x)\cos(x)+\dfrac{1}{x}\sin(x)$

f $\dfrac{2}{x}\cos(x)-\ln(x)\sin(x)-\dfrac{1}{x^2}\sin(x)$

8 **a** $f'(x)=\dfrac{24}{4x-3}$ **b** $f'(2)=\dfrac{24}{5}$ **c** $x=\dfrac{15}{4}$

9 **a** $f'(x)=\dfrac{6x}{x^2-1}$ **b** $f'(2)=4$

c $x=\dfrac{1+\sqrt{5}}{2}$

10 **a** $f'(x)=8x+\dfrac{6(x+1)}{x(x+2)}$ **b** $f'(2)=18.25$

c $x=-1.8363$

11 $g'(1)=2$

12 Proof

7.08

1 **a** $2\ln(x)+c$ **b** $7\ln(x)+c$

c $\dfrac{6}{5}\ln(x)+c$ **d** $\dfrac{4}{7}\ln(x)+c$

e $-\dfrac{8}{11}\ln(x)+c$ **f** $-\dfrac{9}{4}\ln(x)+c$

2 **a** $\ln(x+4)+c,\ x>-4$

b $\ln(x-2)+c,\ x>2$

c $\dfrac{1}{3}\ln(3x+1)+c,\ x>-\dfrac{1}{3}$

d $\dfrac{1}{5}\ln(5x-9)+c,\ x>1\dfrac{4}{5}$

e $\dfrac{11}{7}\ln(7x-9)+c,\ x>1\dfrac{2}{7}$

f $\dfrac{13}{4}\ln(4x-1)+c,\ x>\dfrac{1}{4}$

g $-3\ln(2x-5)+c,\ x>\dfrac{5}{2}$

h $-7\ln(x-3)+c,\ x>3$

3 **a** $x+\ln(x)+c,\ x>0$

b $2x^2-3\ln(x)-\dfrac{1}{x}+c,\ x>0$

c $5\ln(x)+x^2+\dfrac{1}{x}+c,\ x>0$

d $2\ln(x)+\dfrac{4x^3}{3}+\dfrac{1}{x}+c,\ x>0$

e $\dfrac{3x^8}{8}-x^2+15\ln(x)+c,\ x>0$

4 $f(x)=\ln(x-2)+6,\ x>2$

5 $f(x)=-\dfrac{7}{3}\ln(3x-5)+7,\ x>\dfrac{5}{3}$

6 $y=2\ln(x^2+2)+c$

7 $y=\dfrac{1}{2}\ln(x^2-5)+c,\ x^2>5$

8 $\ln(x-2)-\ln(x+2)+c,\ x>2$

7.09

1 **a** $k=\dfrac{1}{20}e^{1.29}$ **b** 59 words

c 238 words **d** 34 (33.3) minutes

e About 2.2 words/minute.

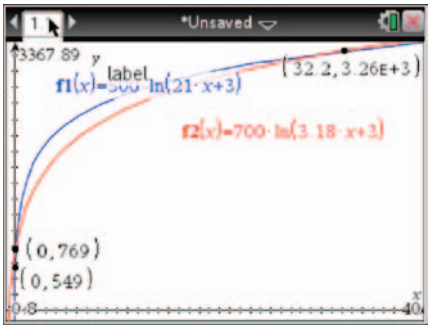
2 **a** 549 black peppered moths

b 3225 black peppered moths

c $t=2.46$ days on 3 January

d $P=700$, $Q=3.189$

e



f 3 February

g About 15 and 21 moths/day respectively.

3 **a** $T(t)=60-4.488\ldots\log(t+1)$

b 168 days

c About 0.027 s/day.

d Over $3\dfrac{1}{2}$ years (1315 days). It is very unlikely his body could stand that kind of continuous training for that length of time.

4 Longer than his lifetime (nearly 900 000 days).

5 **a** $P(t)=1.820\ldots\ln(t+1)+5$ per day.

b About 15 weeks (14.6).

c About 0.36 per week.

d About 0.17 per week.

CHAPTER 7 REVIEW

1 C

2 A

3 D

4 B

5 C

6 E

7 A

8 C

9 $3^4=81$

10 $\log_5(0.04)=-2$

11 About 1.511.

12 2.262

13 $x\approx-0.369$ or $x\approx0.631$

14 $x=\dfrac{-\log(6)-2\log(4)}{3\log(4)-2\log(6)}\approx-7.933$

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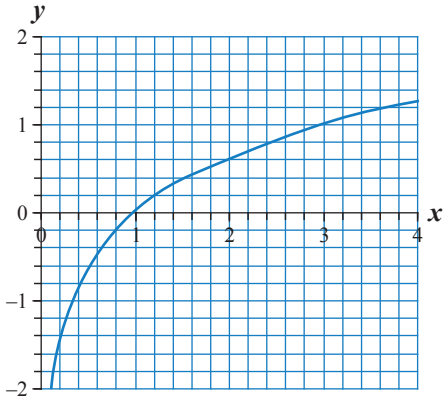
15 1.5

16 $x \approx 0.46$



17 $y = \log_2(x - 1)$

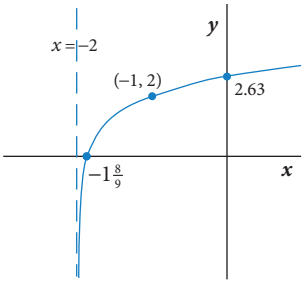
18 a $\dfrac{1}{x\ln(6)}$ b $\dfrac{6x}{3x^2+8}$

c $\dfrac{7(3x^4 - x^3 + 5)}{7x+1} + (12x^3 - 3x^2)\ln(7x+1)$

d $\dfrac{(3x-5)\ln(3x-5) - 3x\ln(x)}{x(3x-5)[\ln(3x-5)]^2}$

19 $\ln(x^3 - 4x + 1) + c$

20



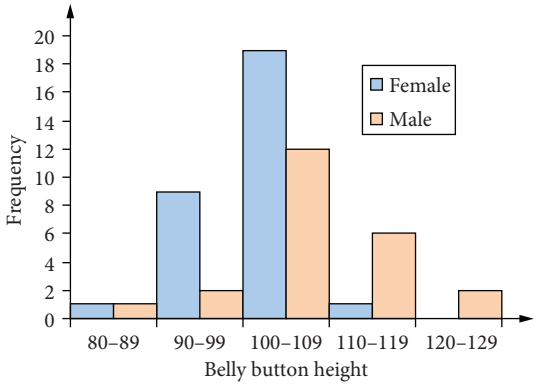
21 About 0.014%.

22 $\dfrac{1}{3}x^3 - 2x^2 + 6x - 10\ln(x+1) + c$

23 a i 2 units ii 6.4 units

b About 0.005 units/minute

INVESTIGATION: BELLY BUTTON HEIGHTS



Mean ≈ 103 cm, SD ≈ 8.1 cm

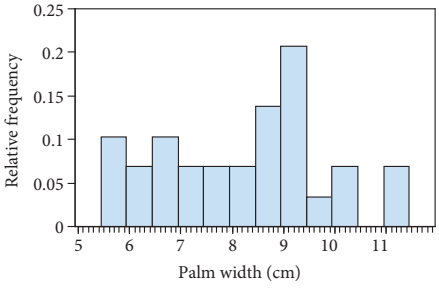
Other answers will vary.

8.01

1 a

Width (cm)	F	R.F.
5.5–5.9	3	0.103
6–6.4	2	0.069
6.5–6.9	3	0.103
7–7.4	2	0.069
7.5–7.9	2	0.069
8–8.4	2	0.069
8.5–8.9	4	0.138
9–9.4	6	0.207
9.5–9.9	1	0.034
10–10.4	2	0.069
10.5–10.9	0	0
11–11.4	2	0.069
Total	29	0.999

b

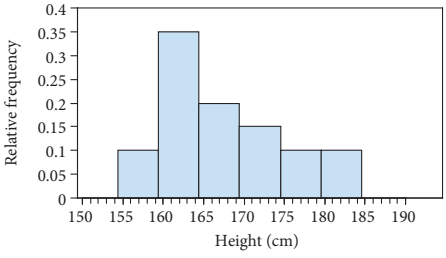


c 0.172 d 0.145

2 a

Height (cm)	Frequency	R.F.
155–159	2	0.1
160–164	7	0.35
165–169	4	0.2
170–174	3	0.15
175–179	2	0.1
180–184	2	0.1

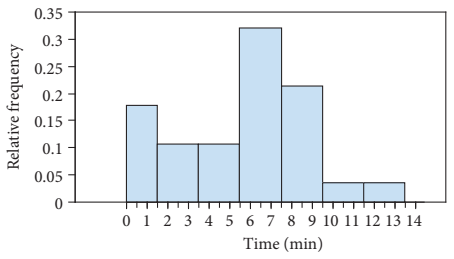
b



c 0.28 d 0.39 e 0.31

3 a

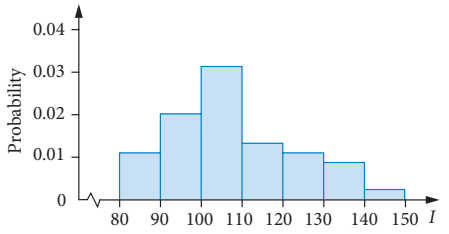
Time (min)	F	R.F.
0–1	5	0.178 571 4
2–3	3	0.107 142 9
4–5	3	0.107 142 9
6–7	9	0.321 428 6
8–9	6	0.214 285 7
10–11	1	0.035 714 3
12–13	1	0.035 714 3

b 

c 0.339 d 0.054 e 0.536

4 a

IQ	F	R.F.
80–89	5	0.113
90–99	9	0.205
100–109	14	0.318
110–119	6	0.136
120–129	5	0.114
130–139	4	0.091
140–149	1	0.023
Total	44	1.000

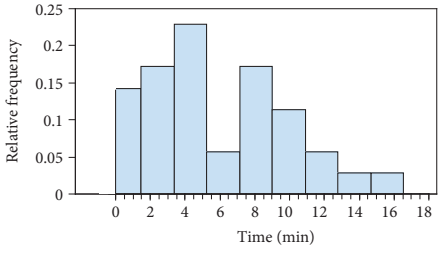


b 0.332 c 0.286 d 107.5

e It doesn't include students of very low IQ. They go to special schools.

5 a

Time (min)	F	R.F.
0–1	5	0.142 857 1
2–3	6	0.171 428 6
4–5	8	0.228 571 4
6–7	2	0.057 142 9
8–9	6	0.171 428 6
10–11	4	0.114 285 7
12–13	2	0.057 142 9
14–15	1	0.028 571 4
16–17	1	0.028 571 4



b 0.429

c They come at intervals of less than 16 minutes.

8.02

1 a Yes b Yes c No, area is not 1

d No, it is negative for (0, 1).

e No, area is not 1

2 a $p(x) = \frac{4}{(x-1)^2}$ b $p(x) = \frac{x^3}{64}$

c $p(x) = \frac{(3-x)(x+3)}{24}$ d $p(x) = 0.005x$

e $p(x) = \frac{2(e^{2x}-1)}{3-2\ln(2)}$

3 a $cdf(x) = 1 - \frac{1}{x}$ b $\frac{1}{2}$ c $\frac{1}{6}$

d $\frac{1}{4}$ e $\frac{1}{12}$

4 a $cdf(x) = \frac{x(x+4)}{140}$ b $\frac{1}{56}$ c $\frac{1}{7}$

d $\frac{19}{28}$ e $\frac{1}{5}$

5 a $cdf(x) = 1.2 - e^{-x}$ b 0

c About 0.737 d 0.8

e About 0.301

6 $f(x) = 0.4 - 0.02x$, it is a straight line with slope -0.02. $f(x) \geq 0$ for $[0, 5]$ and $\int_0^5 f(x)dx = 1$

7 $f(t) = 4e^{-4t}$, it is a decreasing exponential curve. $f(t) \geq 0$ for $[0, 5]$ and $\int_0^\infty f(t)dt = 1$

8 $f(m) = 0.1$, it is a constant function. $f(m) \geq 0$ for $[0, 5]$ and $\int_0^{10} f(m)dm = 1$

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INVESTIGATION: CONTINUOUS PROBABILITY FUNCTIONS FOR MEASUREMENT ERRORS

$p(x) = k(x - 14.8)(x - 15.2)$
$p(x) = -25h(x - 14.8)(x - 15.2)$
$A = \frac{4h}{15}$
$p(x) = -\frac{375}{4}(x - 14.8)(x - 15.2)$
$\int_{14.93}^{15.07} p(x)dx = 0.503\,562$ is a bit less than the triangular function.
Various functions including the normal distribution

8.03

1 **a** $p(x) = 0.05$ **b** $p(x) = \frac{1}{18}$
c $p(x) = 0.1$ **d** $p(x) = 0.1$
e $p(x) = \frac{1}{30}$

2 **a** $p(x) = \begin{cases} \frac{1}{36}x & \text{for } 0 \le x < 6 \\ -\frac{1}{36}(x-12) & \text{for } 6 \le x \le 12 \end{cases}$

b $p(x) = \begin{cases} \frac{1}{64}x & \text{for } 0 \le x < 8 \\ -\frac{1}{64}(x-16) & \text{for } 8 \le x \le 16 \end{cases}$

c $p(x) = \begin{cases} \frac{1}{25}(x-12) & \text{for } 12 \le x < 17 \\ -\frac{1}{25}(x-22) & \text{for } 17 \le x \le 22 \end{cases}$

d $p(x) = \begin{cases} \frac{1}{100}(x-4) & \text{for } 4 \le x < 14 \\ -\frac{1}{100}(x-24) & \text{for } 14 \le x \le 24 \end{cases}$

e $p(x) = \begin{cases} \frac{1}{256}(x-2) & \text{for } 2 \le x < 18 \\ -\frac{1}{256}(x-34) & \text{for } 18 \le x \le 34 \end{cases}$

3 **a** $\frac{1}{3}$ **b** $\frac{1}{6}$ **c** $-\frac{1}{12}$

d $p(x) = \begin{cases} \frac{1}{6}(x-4) & \text{for } 4 \le x < 6 \\ -\frac{1}{12}(x-10) & \text{for } 6 \le x \le 10 \end{cases}$

4 **a** $p(x) = \begin{cases} \frac{1}{10}(x-5) & \text{for } 5 \le x < 7 \\ -\frac{1}{40}(x-15) & \text{for } 7 \le x \le 15 \end{cases}$

b $p(x) = \begin{cases} \frac{1}{12}(x-4) & \text{for } 4 \le x < 8 \\ -\frac{1}{6}(x-10) & \text{for } 8 \le x \le 10 \end{cases}$

c $p(x) = \begin{cases} \frac{1}{15}(x-20) & \text{for } 20 \le x < 23 \\ -\frac{1}{35}(x-30) & \text{for } 23 \le x \le 30 \end{cases}$

d $p(x) = \begin{cases} \frac{1}{150}x & \text{for } 0 \le x < 15 \\ -\frac{1}{50}(x-20) & \text{for } 15 \le x \le 20 \end{cases}$

e $p(x) = \begin{cases} \frac{1}{1400}(x-20) & \text{for } 20 \le x < 60 \\ -\frac{1}{1050}(x-90) & \text{for } 60 \le x \le 90 \end{cases}$

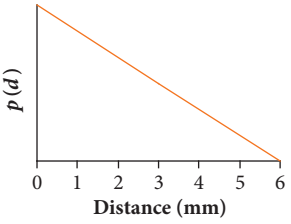
5 **a** $p(t) = \frac{1}{5}$ for $15 \le t < 20$
b 0.8 **c** 0.4

6 **a** 0 and 10 minutes **b** 0.5
c $b(t) = 0.1$ **d** 0.3

7 0.875

8 About 0.848.

9 **a**



b $p(x) = \frac{1}{18}(6 - d)$ **c** $\frac{8}{9}$
d It reduces the size of the target area but he can still have the same probability as with the first dart.

10 **a** $p(t) = \begin{cases} \frac{1}{35}(t-17) & \text{for } 17 \le t < 22 \\ -\frac{1}{63}(t-31) & \text{for } 22 \le t \le 31 \end{cases}$

b $\frac{4}{35}$ **c** About 0.095. **d** $\frac{5}{7}$
e It depends how often she can afford to be late.

8.04

1 **a** $p(x) = \frac{1}{24}$ **b** 15
2 55
3 22
4 20
5 $18\frac{1}{3}$
6 40
7 Proof
8 **a** \$500 000 **b** 0.5
c About \$490 000 (\$490 192.38).

8.05

1 **a** $p(x) = \frac{1}{12}$ **b** $E(X) = 10$

c $Var(X) = 12$, $\sigma = 2\sqrt{3} \approx 3.46$

2 **a** $Var(X) = 33\frac{1}{3}$, $\sigma = \frac{10\sqrt{3}}{3} \approx 5.77$

b $Var(X) = 208\frac{1}{3}$, $\sigma = \frac{25\sqrt{3}}{3} \approx 14.43$

c $Var(X) = 33\frac{1}{3}$, $\sigma = \frac{10\sqrt{3}}{3} \approx 5.77$

d $Var(X) = 133\frac{1}{3}$, $\sigma = \frac{20\sqrt{3}}{3} \approx 11.55$

e $Var(X) = \frac{3}{16}$, $\sigma = \frac{\sqrt{3}}{4} \approx 0.433$

3 **a** $p(x) = \begin{cases} \frac{1}{16}(x-12) & \text{for} \quad 12 \le x < 16 \\ -\frac{1}{16}(x-20) & \text{for} \quad 16 \le x \le 20 \end{cases}$

b $E(X) = 16$

c $Var(X) = 2\frac{2}{3}$, $\sigma = \frac{2\sqrt{6}}{3} \approx 1.63$

4 **a** $p(x) = \begin{cases} \frac{1}{625}x & \text{for} \quad 0 \le x < 25 \\ -\frac{1}{625}(x-50) & \text{for} \quad 25 \le x \le 50 \end{cases}$

b $E(X) = 25$

c $Var(X) = 104\frac{1}{6}$, $\sigma = \frac{25\sqrt{6}}{6} \approx 10.21$

5 **a** $Var(X) = 4\frac{1}{6}$, $\sigma = \frac{5\sqrt{6}}{6} \approx 2.04$

b $Var(X) = 121.5$, $\sigma = \frac{9\sqrt{6}}{2} \approx 11.02$

c $Var(X) = 121.5$, $\sigma = \frac{9\sqrt{6}}{2} \approx 11.02$

6 **a** $p(x) = -0.02(x - 10)$ for $0 \le x \le 10$

b $E(x) = 3\frac{1}{3}$

c $Var(X) = 5\frac{5}{9}$, $\sigma = \frac{5\sqrt{2}}{3} \approx 2.36$

7 Proof

8 $Var(X) = \frac{a^2}{18}$

INVESTIGATION: LINEAR TRANSFORMATIONS OF CONTINUOUS RANDOM VARIABLES

$c' = ac + b$ and $d' = ad + b$

The vertical scale has to be multiplied by $\frac{1}{a}$.

$y = ax + b$ so $x = \frac{y-b}{a}$.

$q(y) = \frac{1}{a}p\left(\frac{y-b}{a}\right)$.

8.06

1 **a** $50, 833\frac{1}{3}$ and $\frac{50\sqrt{3}}{3} \approx 28.87$

b $[5, 205]$

c $105, 3333\frac{1}{3}$ and $\frac{100\sqrt{3}}{3} \approx 57.74$

d $E(Y) = 2E(X) + 5$, $Var(Y) = 4\ Var(X)$ and $SD(Y) = 2SD(X)$

2 **a** $40, 33\frac{1}{3}$ and $\frac{10\sqrt{3}}{3} \approx 5.77$

b $[148, 248]$

c $198, 833\frac{1}{3}$ and $\frac{50\sqrt{3}}{3} \approx 28.87$

d $E(Y) = 5E(X) - 2$, $Var(Y) = 25Var(X)$ and $SD(Y) = 5SD(X)$

3 **a** $40, 133\frac{1}{3}$ and $\frac{20\sqrt{3}}{3} \approx 11.55$

b $[8, 16]$

c $12, 5\frac{1}{3}$ and $\frac{4\sqrt{3}}{3} \approx 2.31$

d $E(Y) = 0.2E(X) + 4$, $Var(Y) = 0.04\ Var(X)$ and $SD(Y) = 0.2SD(X)$

4 **a** $7.5, \frac{1}{24}$ and $\frac{\sqrt{6}}{12} \approx 0.2041$

b $[110, 130]$

c $120, 16\frac{2}{3}$ and $\frac{5\sqrt{6}}{3} \approx 4.082$

d $E(Y) = 20E(X) - 30$, $Var(Y) = 400\ Var(X)$ and $SD(Y) = 20SD(X)$

5 **a** $25, 104\frac{1}{6}$ and $\frac{25\sqrt{6}}{6} \approx 10.21$

b $[-5, 145]$

c $70, 937\frac{1}{2}$ and $\frac{25\sqrt{6}}{2} \approx 30.62$

d $E(Y) = 3E(X) - 5$, $Var(Y) = 9\ Var(X)$ and $SD(Y) = 3SD(X)$

6 **a** $105, 204\frac{1}{6}$ and $\frac{35\sqrt{6}}{6} \approx 14.29$

b $[9.5, 16.5]$

c $13, 2\frac{1}{24}$ and $\frac{7\sqrt{6}}{12} \approx 1.429$

d $E(Y) = 0.1E(X) + 2.5$, $Var(Y) = 0.01\ Var(X)$ and $SD(Y) = 0.1SD(X)$

7 $E(Y) = 173$, $Var(Y) = 225$, $SD(Y) = 15$

8 $\mu = 46$, $\sigma = 8\sqrt{2} \approx 11.32$

9 **a** $p(x) = 0.08 - 0.0032x$

b $8\frac{1}{3}, 34\frac{13}{18}$ and $\frac{25\sqrt{2}}{6} \approx 5.89$

c $[200, 400]$

d $p(y) = -0.000\,05(y - 400)$

e $266\frac{2}{3}, 2222\frac{2}{9}$ and $\frac{100\sqrt{2}}{3} \approx 47.14$

f $E(Y) = 8E(X) + 200$, $Var(Y) = 64\ Var(X)$ and $SD(Y) = 8SD(X)$

10 **a** $p(x) = \begin{cases} \frac{1}{1500}(x-40) & \text{for} \quad 40 \le x < 90 \\ -\frac{1}{300}(x-100) & \text{for} \quad 90 \le x \le 100 \end{cases}$

9780170250290

b $76\frac{2}{3}$, $172\frac{2}{9}$ and $\frac{5\sqrt{62}}{3} \approx 13.12$

c [65, 185]

d $p(y) = \begin{cases} -\frac{1}{6000}(y-65) & \text{for} \quad 65 \le y < 165 \\ -\frac{1}{1200}(y-185) & \text{for} \quad 165 \le y \le 185 \end{cases}$

e $138\frac{1}{3}$, $688\frac{8}{9}$ and $\frac{10\sqrt{62}}{3} \approx 26.25$

f $E(Y) = 2E(X) - 15$, $\text{Var}(Y) = 4\,Var(X)$ and $SD(Y) = 2SD(X)$

8.07

1 a $p(x) = 0.125e^{-0.0488(x-28.5)^2}$

b $p(x) = 0.07e^{-0.0154(x-28.5)^2}$

c $p(x) = 0.07e^{-0.0154(x-48.6)^2}$

d $p(x) = 0.005\,11e^{-0.000\,0822(x-246)^2}$

e $p(x) = 166e^{-86\,806(x-0.07)^2}$

2 a $p(x) = 0.0469e^{-0.006\,92(x-163)^2}$, 0.299

b $p(x) = 2.714e^{-0.000\,0231(x-678)^2}$, 0.447

c $p(x) = 0.001\,124e^{-0.000\,003\,97(x-4240)^2}$, 0.734

d $p(x) = 0.221e^{-0.1543(x-6.5)^2}$, 0.162

e $p(x) = 0.0177e^{-0.000\,979(x-74.9)^2}$, 0.537

3 a 0.4893 **b** 0.4904 **c** 0.4542
d 0.0231 **e** 0.3929

4 a 0.0959 **b** 0.2666 **c** 0.2756
d 0.8997 **e** 0.2529

5 a 0.8819 **b** 0.7947 **c** 0.9894
d 0.9452 **e** 0.3159

6 a 0.713 026 **b** 0.872 203 **c** 0.750 704
d 0.021 596 **e** 0.020 157

7 a 0.272 353 **b** 0.962 131 **c** 0.961 594
d 0.223 549 **e** 0.005 359

8 a 0.051 973 **b** 0.5838 **c** 9.67%

9 11.41%

10 0.1333

11 0.2023

12 0.022 75

8.08

1 a 8.914 **b** 14.12 **c** 8.020
d 12.93 **e** 6.793

2 a 208.2 **b** 839.1 **c** 28.26
d 27.87 **e** 107.7

3 a 0.1113 **b** 78.57

4 a 0.363 **b** 1641.99 **c** −104.72
d 882 **e** 50.264 **f** 1333.584
g 148.428 **h** 6974.4

5 a 96.805 **b** 553.86 **c** 456.12
d 324.52 **e** 148.92 **f** 174.5796
g 92.176 **h** 2357.2

6 $Z_E \approx 0.375$ and $Z_M \approx 0.4$, so he did better in Maths Methods.

7 $Z_{IQ} \approx 0.67$ and $Z_H \approx 0.33$, so her IQ is more unusual than her height.

8 First player $P(X > 37) = 0.091 > P(X > 37) = 0.055$ for second player

9 Only 2, using standard normal scores.

10 35

8.09

Note that all answers in this exercise are approximate.

1 a 5.72% **b** 5.72% **c** 0.42%

2 a 0.717 **b** 0.1764

3 a 3 **b** 31

4 a 0.0334 **b** 0.0549 **c** 0.0182
d 0.0807 **e** 0.008 92 **f** 0.952
g 0.048

5 a 0.0995 **b** 0.0457 **c** 0.0752
d 0.0457 **e** 0.0605

6 0.0153, 0.1497

7 21 months

8 a 15

b No, because there is a 12.8% of this result by chance.

9 38.6–41.4 mm

10 Weldon (2%) is more likely than Betterdon (0.3%).

11 a 0.0912 **b** 0.2469

12 a 0.1359 **b** 0.8609

CHAPTER 8 REVIEW

1 B

2 A

3 E

4 E

5 D

6 B

7 B

8 D

9 0.137

10 $p(x) = \frac{4(x^3 - 2x^2 + 2)}{321}$

11 $p(x) = \frac{1}{6}$, $P(2.25 < x < 2.35) = \frac{1}{60}$

12 16

13 58.6, 11.2

14 $p(x) = 0.0767e^{-0.0185(x-76)^2}$

15 English, as $Z_E = 1.52$ and $Z_M = 1.31$

16 a 0.7580 **b** 0.4125

17 About 98 000 km

18 0.2

19 $\frac{5}{36}$

20 0.3765

MIXED REVISION 3

Multiple choice

1 B
2 C
3 C
4 A
5 E
6 A

Short answer

1 $2\frac{2}{3}$
2 $x < \frac{2}{3}$
3 The constant C translates the graph horizontally left ($C > 0$) or right ($C < 0$).
4 About 79.0 kg.

Application

1 41, 57 and 68, to the nearest mark.
2 Proof
3 $y = 3\log_e(x + 1) + 2$
4 a $E = \log_{10}\left(\frac{C_2}{C_1}\right)$
  b $E = 0.42$ kilocalories per gram molecule
  c $E = 1.09$ kilocalories per gram molecule

INVESTIGATION: POLITICAL POLLS

They discard those who did not answer or who answered 'don't know'. It is generally reasonable, but may introduce some non-response bias.
They allocate their preference partly to each of the two parties based on historical preferences of people who voted that way in the past. It is reasonable because it produces reasonable results.
The Reader's Digest poll went to a selected group: Reader's Digest readers, and these were not typical of the whole population. There was also considerable non-response bias and the poll was conducted some time before the election.

9.01

1 a Pies produced; mass and colour; mean = 103.2 g, range = 12 g, all even colour
  b Guests at hotel; room service speed, room service food quality; speed 3.5, food quality 3.7, on averaged scale.
  c Brisbane passenger buses; number of passengers; mean ≈ 29.7 passengers, range = 33 passengers.
  d People catching planes from Cairns; time to reach airport; mean ≈ 16.7 minutes, range = 40 minutes.
  e Checkout operators; number of errors, correctness of till totals, value processed; 27% made 5 errors, 20% incorrect till totals, 13% processed $10 000
2 B
3 B
4 A
5 a Reasonably fair; Selection bias, only current opening hours customers polled; Non-response bias, people who don't care may not respond.
  b Biased; Selection bias; only students from this school; Recall/reporting bias, non-secret system so students may not report low amounts.
  c Biased; Self-selection bias, only people who are very interested will respond and it could be open to manipulation by friends of the contestants.
  d Biased; Non-response bias, people who have positive views or don't care are unlikely to attend the meetings; Selection bias, local meetings.
  e Biased; Selection bias, the reviewer decides on restaurants; Reporting bias, restaurant's knowledge of the critic may change its response to the critic.
6 a Students at the school who are legally able to work.
  b Proportion with part-time jobs, hours worked.
  c Population will change as more students reach working age during the year; Reporting bias, students may under-report hours to the school.
7 Selection bias, students later in the alphabet (the last 125) are not chosen, students from the same family cannot be selected; Non-response bias, absent students may be those who find conduct of the test poor.
8 a Selection bias, only one year group.
  b Selection bias, only students who use the resource centre.
  c Selection bias, only one year group.
  d Self-selection bias.
  e Possible selection bias as students in the same family cannot be surveyed.
  f Self-selection bias.
  g Possible selection bias as classes may not be the same size. Self-selection bias.
  h Possible selection bias as year groups may not be the same size.
  i Self-selection bias.
  j Selection bias, as only early-birds will be selected.

The best methods are likely to be (in order) **e**, **g** and **h.**

INVESTIGATION: PSEUDO-RANDOM NUMBERS

$x_n = 20x_{n-1} \bmod 37$, starting with 5, repeats after 37 numbers

$x_n = 12x_{n-1} \bmod 19$, starting from 7, repeats after 6 numbers

The repeat cyle for the Blum Blum Shub generator is very long.

One method uses the Blum Blum Shub generator where the receiver generates the encoding generator number and sends it to the sender. The sender then encodes using this generator number, but cannot decode. The receiver can decode the message.

Breaking the code takes a lot of computer power.

INVESTIGATION: RANDOM AND NON-RANDOM SAMPLES

Results will vary, but there is usually some attempt at randomisation.

9.02

1 **a** 50, 42, 76, 44, 43, 79, 47, 40
b 374 995, 380 868, 139 324, 347 249, 105 078, 110 178, 138 491, 336 824
c 7, 0, 2, 6, 4, 8, 3, 9
d 66, 39, 57, 63, 53, 32, 61, 67

2 **a** 4, 3, 8, 13, 11, 48, 36, 32, 1, 19
b 576, 689, 631, 566, 682, 644
c 1068, 1602, 1729, 1535, 1696, 2155, 1566, 2079
d 344, 866, 759, 507, 466, 928, 462, 672, 257, 916, 855, 553

3 Answers will vary.

4 **a** 3 men in board shorts, 2 men in briefs, 4 women in bikinis and 1 woman in a one-piece.
b 7 soft-centred, 5 hard-centred, 3 liquid-centred and 4 nutty-centred chocolates (= 19). An extra hard-centred would be chosen if exactly 20 were needed.
c 2 fifteen-year-olds, 11 sixteen-year-olds, 3 seventeen-year-olds and 1 eighteen-year-old (= 17). If exactly 16 were required, you would choose only one 15-year-old.
d 11 assembly workers, 3 office staff and 1 supervisor.

5 **a** 28, 42, 56, 70, 84, 98, 112, 126, 140, 12
b 216, 244, 272, 300, 105, 133, 161, 189
c 64, 92, 120, 148, 176, 204, 232, 260, 288, 316, 344, 372, 400, 1, 29
d 1472, 1647, 1822, 1997, 2172, 2347, 2522, 2697, 1296

6 **a** Quota **b** Convenience
c Judgement **d** Purposive
e Convenience **f** Quota
g Judgement **h** Purposive

7 5130, 5205, 5280, 5355, 5430, 5505, 5580, 5655, 4985, 5060

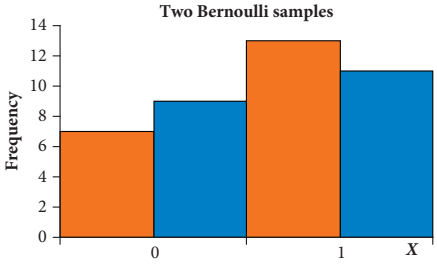
8 **a** Administration: males 1, females 3; Factory: males 8, females 6
b Administration 4, factory 14

9 **a** NSW 160 or 161 Vic. 124, Qld 101, SA 36, WA 54, Tas. 11, NT 5, ACT 8
b Men: NSW 80, Vic. 61, Qld 50, SA 18, WA 27, Tas. 6, NT 3, ACT 4
Women: NSW 81, Vic. 63, Qld 50, SA 18, WA 27, Tas. 6, NT 2, ACT 4

10 Numbers 205, 224, 243, 262, 281, 300, 319, 338, 357, 376, 9, 28, 47, 66, 85, 104, 123, 142, 161, 180

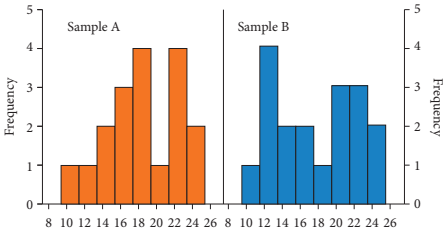
9.03

1 **a**



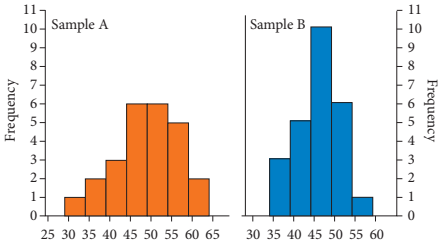
b $\mu_A = 0.65$, $\sigma_A \approx 0.477$, $\mu_B = 0.55$, $\sigma_B \approx 0.497$; Sample A has a higher mean and less spread than sample B.

2 **a**



b $\mu_A \approx 18.4$, $\sigma_A \approx 3.84$, $\mu_B \approx 17.7$, $\sigma_B \approx 4.51$; Sample A has a higher mean and less spread than sample B.

3 **a**



b $\mu_A \approx 49.4$, $\sigma_A \approx 7.61$, $\mu_B \approx 46.2$, $\sigma_B \approx 5.42$; Sample A has a higher mean and more spread than sample B.

Specific answers for questions **4**–**10** will vary, but the general conclusions will be the same.

4 a The dot plots will be similar in overall shape.
b The means will be close to 17.5 and the standard deviations will be close to 2.5.
c The samples will probably have different frequencies but they will have similar patterns.
5 a The dot plots will be different.
b The means will be close to 0.2 and the standard deviations will be close to 0.4.
c The samples will have different appearances.
6 a The dot plots will be different.
b The means will be close to 15 and the standard deviations will be close to 6.
c The samples will have different appearances.
7 a The dot plots will be different.
b The means will be close to 50 and the standard deviations will be close to 10.
c The samples will have different appearances.
8 a The means will be close to 9 and the standard deviations will be close to 2.
b The means will be close to 9 and the standard deviations will be close to 2.
c The difference between the means and the standard deviations for the samples of 64 items will be less than the difference for the samples of 9 items.
9 a The means will be close to 40 and the standard deviations will be close to 3.
b The means will be close to 40 and the standard deviations will be close to 3.
c The difference between the means and the standard deviations for the samples of 64 items will be less than the difference for the samples of 16 items.
10 a The means will be close to 40 and the standard deviations will be close to 8.
b The means will be close to 40 and the standard deviations will be close to 8.
c The difference between the means and the standard deviations for the samples of 81 items will be less than the difference for the samples of 9 items.

9.04

1 0.045
2 0.15
3 0.064
4 a 0.225 b 0.421 c 0.048
5 0.214
6 0.068
7 The cheaper paintings may be under-represented in a commercial gallery.
8 a 0.343
b Older caravan and motor home drivers may be over-represented in Queensland in winter as 'grey nomads' tend to travel north in winter.

9.05

1 a $E(\hat{p}) = 0.2$, $Var(\hat{p}) = 0.0032$, $SD(\hat{p}) \approx 0.057$
b $E(\hat{p}) = 0.7$, $Var(\hat{p}) = 0.0084$, $SD(\hat{p}) \approx 0.092$
c $E(\hat{p}) = 0.81$, $Var(\hat{p}) \approx 0.0013$, $SD(\hat{p}) \approx 0.036$
d $E(\hat{p}) = 0.22$, $Var(\hat{p}) = 0.0022$, $SD(\hat{p}) \approx 0.046$
2 $E(\hat{p}) = 0.\overline{3}$, $SD(\hat{p}) \approx 0.0703$
3 $E(\hat{p}) = 0.45$, $SD(\hat{p}) \approx 0.035$
4 $E(\hat{p}) = \frac{1}{6}$, $SD(\hat{p}) \approx 0.068$
5 $E(\hat{p}) = \frac{3}{13}$, $SD(\hat{p}) \approx 0.042$
6 About 0.6.
7, 8 Proofs

INVESTIGATION: CABBAGE MOTHS

As the sample size is increased, the sample means, standard deviations and proportions get closer to the class average, which becomes more stable.

9.06

1 a–c Results will vary.
d As n increases, the distribution more closely approximates a normal distribution.
2 a–c Results will vary.
d As n increases, the distribution more closely approximates a normal distribution.
3 a–c Results will vary.
d As p gets closer to 0.5, the distribution more closely approximates a normal distribution.
4 a–c Results will vary.
d As p gets closer to 0.5 and n increases, the distribution more closely approximates a normal distribution.
5 a–c Results will vary.
d As n increases, the distribution more closely approximates a normal distribution; the mean gets closer to 25 and the standard deviation decreases from about 2 to 0.8 to 0.5 compared to the original 7.2.
6 a–c Results will vary.
d As n increases, the distribution more closely approximates a normal distribution;the mean gets closer to the original distribution mean and the ratio of the standard deviation of the sampling distribution to that of the original distribution decreases.
7 a–c Results will vary.
d As n increases the distribution more closely approximates a normal distribution, the mean gets closer to 55 and the standard deviation decreases from about 5 to about 2.4 to about 1 compared to the original 12

8 **a, b, c, d** Results will vary.
e The distribution becomes more like a normal distribution.
9 **a, b, c, d** Results will vary.
e The distribution becomes more like a normal distribution.
10 **a, b, c, d** Results will vary.
e The mean gets closer to 184 cm and the standard deviation decreases from about 2.3 to about 1.3.

INVESTIGATION: NORMALISED SAMPLE PROPORTIONS

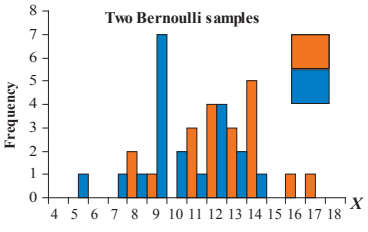
The normalised graph becomes more and more like a normal distribution as the sample size increases, but the width decreases.

9.07

1 10.8%
2 31.2%
3 4.78%
4 0.220
5 0.000169
6 0.1108
7 **a** 0.026 26 **b** 0.9988 **c** 0.008 939
8 0.2585

CHAPTER 9 REVIEW

1 B
2 A
3 C
4 C
5 D
6 E
7 **a** People who follow AFL, proportion most disliking each team, proportion disliking Collingwood = 0.25.
b Houses in the area, number of bedrooms, mean ≈ 3, range 1 to 5.
8 **A**
9 Answers will vary.
10 3 welders, 1 boilermaker, 2 sheet metal workers and 4 labourers.
11 17 113, 18 527, 19 942, 10 042, 11 456, 12 870, 14 285, 15 699
12 **a**



b $\mu_A = 12.4 > \mu_B = 10.1$, but $\sigma_A \approx 2.29 \approx \sigma_B \approx 2.19$; this is also shown by the graphs.
13 0.75
14 **a** 0.375
b This is only one shop, and since it is a sale, the estimate may be low.
15 $\hat{p} = 0.59$ and $\sigma \approx 0.0898$
16 **a** The distribution becomes more like a normal distribution.
b The distribution gets less like a normal distribution.
17 About 71%.
18 **a** Convenience sampling, selection bias, recall/reporting bias.
b Convenience sampling, as not everyone has a landline, some selection bias; probably some completion bias (reasons) and probably non-response bias.
19 **a** Answers will vary.
b Answers will vary but they should both be between about 18 and 22. The standard deviation is likely to be between about 2 and 4.
c The theoretical mean is $\frac{15+25}{2} = 20$ and the theoretical SD $= \frac{25-15}{2\sqrt{3}} \approx 2.89$. The mean is close to the theoretical mean but the standard deviation could vary considerably.
20 10 (10.8)
21 Australia: about 0.82; Britain: about 0.0036.

INVESTIGATION: POINT AND INTERVAL ESTIMATES IN THE MEDIA

Results will vary.

10.01

1 **a** Point **b** Interval **c** Interval **d** Point
2 $p \approx 0.71$, $\sigma^2 \approx 0.0026$, $\sigma \approx 0.051$
3 $p \approx 0.45$, $\sigma^2 \approx 0.0062$, $\sigma \approx 0.079$
4 $p \approx 0.60$, $\sigma^2 \approx 0.0028$, $\sigma \approx 0.053$
5 $p \approx 0.32$, $\sigma^2 \approx 0.0044$, $\sigma \approx 0.066$
6 0.57 to 0.71
7 0.55 to 0.81
8 0.49 to 0.76
9 0.0222
10 42

10.02

1 **a** About 98.8%. **b** $\alpha \approx 0.997$
c About 0.683. **d** $\alpha \approx 0.993$
2 **a** $E \approx 2.33$ **b** $E \approx 1.44$
c $E \approx 2.58$ **d** $E \approx 1.15$

3 **a** About $-1.64 \leq z < 1.64$
b About $-2.81 \leq z < 2.81$
c About $-1.96 < z < 1.96$
d About $-2.58 \leq z < 2.58$

4 **a** About 0.988 **b** $\alpha \approx 0.866$
c $\alpha \approx 0.683$ **d** About 99.7%.

5 The confidence level is the area under the curve. To get a larger area under the curve, you need a wider interval. The margin of error is half the interval length, so you cannot have a high confidence level and a low margin of error.

10.03

1 **a** 0.70 **b** 0.065 **c** (0.57, 0.83)

2 **a** $E \approx 0.104$ **b** $E \approx 0.127$
c $E \approx 0.108$ **d** $E \approx 0.061$

3 **a** (0.044, 0.156) **b** (0.616, 0.884)
c (0.174, 0.466) **d** (0.514, 0.706)

4 (0.850, 0.936)

5 (0.415, 0.562)

6 (0.013, 0.107)

7 About 2.4%.

8 0.049, 0.075, 0.082, 0.075 and 0.049 so the error margin increases as the sample proportion gets closer to 0.5 for the same size sample and confidence level.

INVESTIGATION: DICE SIMULATION OF CONFIDENCE INTERVALS

For 20 rolls the proportion of high rolls is likely to be between 0.06 and 0.60 and in most cases will be close to the centre (0.33).
You are likely to have between 0 and 2 confidence intervals that do not contain 0.33.
The class average is likely to be between 5% and 15% not containing 0.33.

10.04

1–5 Answers will vary, but pooled results should approach the proportion given by the confidence level in each case.

10.05

1 About 1300 (1262).

2 About 300 (271).

3 About a million (960 400).

4 About 70 (68).

5 About 4000 (3981).

6 About 1000 (1067).

7 About 56%.

8 About 85%.

CHAPTER 10 REVIEW

1 C

2 E

3 C

4 A

5 D

6 D

7 B

8 0.64–0.74

9 98.8% (0.98758…)

10 1.75

11 **a** 0.275 **b** 0.071 **c** (0.14, 0.41)
d The proportion of Year 12 students having a part-time job has a probability of 95% of being in the interval (0.14, 0.41).

12 **a** About 0.065. **b** (0.685, 0.815)

13 **a** Answers will vary, but should be of the form {0,1,1,1,0,0,1,1,1,1,1,1,1,1,1,0,1,1,1,0,0,1,1,1,0, 1,0,0,1}.
b In this case, it is (0.54, 0.86); they should be about the same width.
c Mention that in most cases (95%), 0.67 will be inside the interval, whether it is or not.

14 About 500 (454).

15 **a** That the sample proportion is 0.5.
b About 110 (106).

16 244

17 **a** 0.0209 **b** 1.440 **c** (0.29, 0.35)

18 **a** About 60 (61). **b** About 11 minutes.

MIXED REVISION 4

Multiple choice

1 B

2 D

3 B

4 C

5 E

6 D

Short answer

1 6 Year 7s, 7 Year 8s, 7 Year 9s, 8 Year 10s, 6 Year 11s and 5 Year 12s making 39 altogether, rather than 40 because of rounding errors. To get 40, an extra Year 8 student would be chosen.

2 (0.24, 0.36)

3 **a** $\frac{1}{12}$, $\frac{1}{6}$ and $\frac{1}{4}$ respectively.
b About 0.001 060, 0.001 93 and 0.002 60 respectively.
c About 0.0326, 0.0439 and 0.0510 respectively.

4 **a** 0.0162 **b** 2.326 **c** (91.2%, 98.8%)

Application

1 99%

2 About 660 (664).

3 a Simple random sampling of voters, possible interviewer bias if they were not given the same script.

b Convenience/cluster sampling, possible selection bias.

c Quota sampling, selection bias and some design bias as the question is one-sided.

d Simple random sampling of those with mobile phones, non-response bias.

4 About 0.88.